A comparative study on modal-based finite element model updating approaches using noisy measurements

D. Zhu, X. Dong & Y. Wang
School of Civil and Environmental Engineering, Georgia Institute of Technology, Atlanta, GA, USA

ABSTRACT: In order to identify more accurate finite element (FE) model parameters for an as-built structure, experimental data collected from the actual structure can be used to update the parameter values. In practice, experimental data is inevitably contaminated with measurement noise, which may lead to inaccurate FE model updating. This research compares the performance of two model updating approaches under noisy measurements. The first approach minimizes the difference between experimental and simulated modal properties, such as natural frequencies, mode shapes, and modal flexibilities. The second approach minimizes modal dynamic residuals from the generalized eigenvalue equation involving stiffness and mass matrices. Numerical study of a 6-degree-of-freedom spring-mass structure is performed through Monte Carlo simulation that generates noise-contaminated modal properties. Performance of the two model updating approaches is compared.

1 INTRODUCTION

During the past few decades, many efforts have been devoted to developing accurate finite element (FE) models. However, owing to the complexity of civil structures, structural behavior predicted by FE models (built according to design drawings) is usually different from behavior of actual structures in the field. The discrepancy is mainly caused by limited accuracy of FE modeling. For example, in an actual structure, support conditions are far more complicated than ideal hinges, fixed ends, or rollers commonly used in modeling and design. Besides, most structural components may deteriorate over time. As a result, an FE model based on original structural drawings does not accurately reflect the deteriorated structure.

To improve the model accuracy, FE model updating can be performed based on sensor measurement from the actual structure. In the past few decades, various FE model updating methods have been developed and practically applied (Friswell & Mottershead 1995). Many of these methods utilize modal analysis results from field testing. Selected structural parameters are updated by solving an optimization problem. One major category of model updating methods minimizes the difference between experimental and simulated modal properties. This category will be referred as modal property difference approach. For example, FE model updating using changes in mode shapes and frequencies was applied for damage assessment of a reinforced concrete beam (Teughels *et al.* 2002). Jaishi & Ren (2006) proposed an objective function consisting of changes in frequencies, modal assurance criterion (MAC) related functions, and modal flexibility for updating the model of a beam structure. Another category of model updating methods will be referred as modal dynamic residual approach, which minimizes modal dynamic residuals from the generalized eigenvalue equation involving stiffness and mass matrices. For example, Farhat and Hemez (1993) proposed an iterative least-square (LSQ) algorithm to update element stiffness and mass properties by minimizing the norm of modal dynamic residuals. The method was validated through simulation of a 2D truss structure and a cantilever beam. Abdalla *et al.* (2000) formulated a linear matrix inequality problem that minimizes the change in stiffness matrix (from initial estimate) under constraints on the magnitude of modal dynamic residuals.

Despite intensive research efforts, FE model updating approaches have not been widely applied in practice. One of the major obstacles lies in the fact that experimental data is inevitably contaminated with certain measurement noise. The noisy data produces uncertainties in model updating results. Researchers have investigated noise effect on some FE model updating approaches. For example, Ahmadian *et al.* (1998) investigated the regularized modal dynamic residual approach for FE model updating using noisy measurements. Hua *et al.* (2012) presented a numerical study of noise effect on the modal property difference approach through Monte Carlo simulation.

This research compares the noise effect on the two aforementioned modal-based model updating approaches, i.e. modal property difference approach and modal dynamic residual approach. The rest of the paper is organized as follows. The formulations of both model updating approaches are presented first. Numerical investigation on a 6-DOF spring-mass structure through Monte Carlo simulation is then described, where artificial Gaussian noise is introduced into “measured” natural frequencies and mode shapes to be used for model updating. Performance of both model updating approaches is compared. Finally, a summary and discussion are provided.

2 MODEL UPDATING APPROACHES

For a linear structural system, the system stiffness can be updated as:

$$\mathbf{K} = \mathbf{K}_0 + \sum_{i=1}^{n_\alpha} \alpha_i \mathbf{K}_{0,i} \tag{1}$$

where $\mathbf{K}$ denotes the overall structural stiffness matrix; $\mathbf{K}_0$ is the constant initial stiffness matrix estimated prior to model updating; n_α is the total number of updating parameters; $\alpha_i \ (i=1,\ldots,n_\alpha)$ represent stiffness parameters to be updated; $\mathbf{K}_{0,i}$ is the constant sensitivity matrix that corresponds to the contribution of the associated updating parameter α_i. In this preliminary research, it is assumed accurate structural mass matrix is known.

2.1 *Modal property difference approach*

In modal property difference approach, an optimization problem is usually formulated to minimize the difference between experimental and simulated natural frequencies and mode shapes (Eq. (2)). Compared with experimental properties obtained from dynamic testing in the field, the simulated properties are generated by an FE model.

$$\underset{\alpha_1,\cdots,\alpha_{n_\alpha}}{\text{Minimize}} \ \sum_{i=1}^{m} w_i^2 \left\{ \left(\frac{\omega_{\text{FE},i} - \omega_{\text{exp},i}}{\omega_{\text{exp},i}} \right)^2 + \left(\frac{1-\sqrt{\text{MAC}_i}}{\sqrt{\text{MAC}_i}} \right)^2 \right\} \tag{2}$$

where m denotes the number of available experimental modes being used for model updating; $\omega_{\text{FE},i}$ and $\omega_{\text{exp},i}$ represent the i-th simulated (from FE model) and experimental natural frequencies, respectively; MAC_i represents the modal assurance criterion evaluating the difference between the i-th simulated and experimental mode shapes (i.e. $\boldsymbol{\psi}_{\text{FE},i}$ and $\boldsymbol{\psi}_{\text{exp},i}$); w_i is the weighting factor of the i-th measured mode. Since different modes may have different accuracies (in practice, lower-frequency modes tend to be more accurate), larger weighting factors can be assigned to lower-frequency modes. In this research, a nonlinear least-square optimization solver, ‘lsqnonlin’ in MATLAB toolbox (MathWorks Inc. 2005), is adopted to numerically solve the optimization problem (Eq. (2)) in modal property difference approach. The optimization solver seeks a minimum of the objective function in Eq. (2) through Levenberg-Marquardt algorithm (Moré 1978), which adopts a search direction interpolated between the Gauss-Newton direction and the steepest descent direction. At every search step, according to Eq. (1) the solver reassembles the stiffness matrix $\mathbf{K}$ using updated values for parameters $\alpha_i \ (i=1,\ldots,n_\alpha)$. The generalized eigenvalue problem between mass matrix $\mathbf{M}$ and updated stiffness $\mathbf{K}$ is solved for $\omega_{\text{FE},i}$ and $\boldsymbol{\psi}_{\text{FE},i}$. The objective function in Eq. (2) is then evaluated for determining search direction, along which a temporary optimal solution is obtained for next search step.

2.2 *Modal dynamic residual approach*

For comparison with modal property difference approach, a modal dynamic residual approach is studied using a regularized least square optimization formulation.

$$\underset{\alpha_1,\cdots,\alpha_{n_\alpha}}{\text{Minimize}} \ \sum_{i=1}^{m} w_i^2 \left\| \left(\mathbf{K} - \omega_{\text{exp},i}^2 \mathbf{M}\right) \boldsymbol{\psi}_{\text{exp},i} \right\|^2 + \lambda^2 \left\| \boldsymbol{\alpha} \right\|^2 \tag{3}$$

where $\|\bullet\|$ denotes Euclidean norm (2-norm); $\omega_{\text{exp},i}$ and $\boldsymbol{\psi}_{\text{exp},i}$ denote the i-th modal frequency and mode shape from experimental data; $\mathbf{M}$ denotes structural mass matrix; w_i is the weighting factor of the i-th measured mode; $\boldsymbol{\alpha}$ contains stiffness parameters to be updated $(\alpha_i, \ i=1,\ldots,n_\alpha)$; λ is the regularization parameter, which balances the weightings between modal dynamic residuals and parameter changes from initial FE model. The regularized objective helps to limit erratic changes in the system updating parameters, particularly when measurement noise is present. Although the selection of regularization parameter λ deserves in-depth study, in this preliminary research, a constant regularization parameter is adopted.

During dynamic testing in the field, usually not all DOFs can be measured by sensors. This indicates that only incomplete mode shapes can be directly obtained from experimental data. Therefore, in addition to updating parameters α_i, part of $\boldsymbol{\psi}_{\exp,i}$ is unknown in the optimization formulation in Eq. (3). In this case, modal expansion techniques can be adopted to obtain mode shapes at unmeasured DOFs. In detail, this study follows an iterative linearization procedure (Farhat & Hemez 1993) for efficiently solving the optimization problem. Each iteration includes two steps:

Step (i) Modal Expansion

In Step (i), stiffness parameters in $\boldsymbol{\alpha}$ are treated as constant. The parameter values are either based on initial estimation, or from model updating results in the last iteration. Modal expansion can be performed to obtain the unknown part of each experimental mode shape vector $\boldsymbol{\psi}_{\exp,i}$. To lighten notation, $\boldsymbol{\psi}_{\exp,i}$ is simplified as $\boldsymbol{\psi}_i$ in the rest of this section. The modal expansion is performed as:

$$\boldsymbol{\psi}_{i,U} = -\left(\mathbf{A}_{UU}{}^{-1}\mathbf{A}_{MU}\right)\boldsymbol{\psi}_{i,M} \tag{4}$$

where subscripts M and U represent the measured and unmeasured DOFs, respectively; $\boldsymbol{\psi}_{i,M}$ and $\boldsymbol{\psi}_{i,U}$ represent the measured and unmeasured entries of the i-th mode shape vector. The expansion matrix $\left(\mathbf{A}_{UU}{}^{-1}\mathbf{A}_{MU}\right)$ comes from the generalized eigenvalue problem in the structural dynamics:

$$\begin{bmatrix} \mathbf{A}_{MM} & \mathbf{A}_{MU} \\ \mathbf{A}_{UM} & \mathbf{A}_{UU} \end{bmatrix} = \mathbf{A}_i = \mathbf{D}_i{}^{\mathrm{T}}\mathbf{D}_i \tag{5}$$

where

$$\mathbf{D}_i = \left(\mathbf{K} - \omega_i^2\mathbf{M}\right) \tag{6}$$

Step (ii) Parameter Updating

Using the expanded complete mode shapes from Step (i), the stiffness parameters $\boldsymbol{\alpha}$ can be obtained by solving the optimization problem (Eq. (3)) in regularized least square form. Eq. (3) can be rewritten as follows:

$$\left(\mathbf{B}^{\mathrm{T}}\mathbf{B} + \lambda^2\mathbf{I}\right)\boldsymbol{\alpha} = \mathbf{B}^{\mathrm{T}}\mathbf{r} \tag{7}$$

where

$$\mathbf{B} = \begin{bmatrix} w_1\mathbf{K}_{0,1}\boldsymbol{\psi}_1 & w_1\mathbf{K}_{0,2}\boldsymbol{\psi}_1 & \cdots & w_1\mathbf{K}_{0,n_\alpha}\boldsymbol{\psi}_1 \\ w_2\mathbf{K}_{0,1}\boldsymbol{\psi}_2 & w_2\mathbf{K}_{0,2}\boldsymbol{\psi}_2 & \cdots & w_2\mathbf{K}_{0,n_\alpha}\boldsymbol{\psi}_2 \\ \vdots & \vdots & \ddots & \vdots \\ w_m\mathbf{K}_{0,1}\boldsymbol{\psi}_m & w_m\mathbf{K}_{0,2}\boldsymbol{\psi}_m & \cdots & w_m\mathbf{K}_{0,n_\alpha}\boldsymbol{\psi}_m \end{bmatrix} \tag{8}$$

$$\mathbf{r} = \begin{Bmatrix} -w_1\left(\mathbf{K}_0 - \omega_1^2\mathbf{M}\right)\boldsymbol{\psi}_1 \\ -w_2\left(\mathbf{K}_0 - \omega_2^2\mathbf{M}\right)\boldsymbol{\psi}_2 \\ \vdots \\ -w_m\left(\mathbf{K}_0 - \omega_m^2\mathbf{M}\right)\boldsymbol{\psi}_m \end{Bmatrix} \tag{9}$$

The updated model can be used as an initial model again in Step (i). Updating process is performed iteratively for higher accuracy until updating parameters converge.

3 NUMERICAL EXAMPLES

To compare the modal property difference approach and the modal dynamic residual approach, a 6-DOF spring-mass model is simulated (Figure 1). Table 1 summarizes the stiffness properties of the model. The springs are assigned with different stiffness values, and the masses blocks are identically set to 6 kg. For simplicity, the natural frequencies and mode shapes are directly obtained from solving generalized eigenvalue equation, and used as "experimental" results for model updating.

Two measurement cases are studied. Case 1 assumes every node is measured, and thus, complete experimental mode shapes can be obtained. Because modal expansion is not needed, optimization formulation of the modal dynamic residual approach becomes a simple least-square problem on stiffness parameters k_i. The problem can be solved without iteration. Case 2 assumes partial DOFs are measured, where the iterative process is necessary in modal dynamic residual approach. For both cases, random errors in normal distribution are assigned to every natural frequency and mode shape vector.

$$\tilde{\boldsymbol{\psi}}_{\exp,i} = \boldsymbol{\psi}_{\exp,i} + \boldsymbol{\zeta}_i \tag{10}$$

$$\tilde{\omega}_{\exp,i} = \omega_{\exp,i}\cdot\left(1+\xi_i\right) \tag{11}$$

where $\boldsymbol{\psi}_{\exp,i}$ denotes the normalized i-th mode shape with maximum entry magnitude equal to 1; $\boldsymbol{\zeta}_i$ denotes a zero-mean Gaussian random vector. Assuming the first mode is more reliable, the stand-

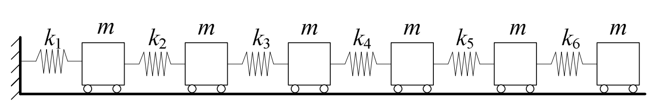

Figure 1. 6-DOF spring-mass structure

Table 1. Structural properties

Property	k_1	k_2	k_3	k_4	k_5	k_6
Spring stiffness (10^4N/m)	2.80	3.15	2.45	3.50	4.20	3.85

ard deviation of each entry in noise vector ζ_1 is set as 0.01 for the first mode. A standard deviation of 0.03 is assigned to entries in other noise vectors for all higher-frequency modes. In Eq. (11), ξ_i denotes the relative random error in normal distribution (zero mean) for the i-th frequency. Similarly, the standard deviation of error term ξ_1, for the first mode, is set as 0.01, while a standard deviation of 0.03 is assigned for all higher modes. According to the noise level, the weighting parameter w_i (Eq. (2) and (3)) is assigned to be 2 for the first mode, and 1 for all other modes. Regularization parameter λ (Eq. (3)) is set to 1,000 in all simulations when noise is present.

For both model updating approaches, Monte Carlo simulation is performed for J = 10,000 runs to generate J sets of "noisy" modal properties. The noisy modal properties are used as experimental data input to conduct model updating. For consistency in comparing the two model updating approaches, at each run, the random seed in MATLAB is fixed to generate the same J sets of noisy modal properties for both approaches. The root mean square (RMS) of the relative difference between updated and actual parameters is calculated to evaluate the updating performance for each parameter α_i.

$$\mathrm{RMS}_i = \sqrt{\frac{1}{J}\sum_{j=1}^{J}\left(\frac{\alpha_{i,j}^{\mathrm{upd}} - \alpha_i^{\mathrm{act}}}{\alpha_i^{\mathrm{act}}}\right)^2} \quad (12)$$

where α_i^{act} denotes the actual value of the i-th updating parameter (Table 1), and $\alpha_{i,j}^{\mathrm{upd}}$ represents the updated optimal value of the i-th parameter in the j-th run. In this numerical study, the updating parameters α_i simply refer to stiffness parameters k_i.

3.1 *Case 1: complete measurement*

In this case, all the DOFs are measured. For reference, both model updating approaches are first applied when no noise is added to experimental modal

Table 2. Model updating results

Updating parameter		k_1	k_2	k_3	k_4	k_5	k_6	Avg. error (%)
		(10^4N/m)						
Initial Value		3.50	3.50	3.50	3.50	3.50	3.50	17.45
Modal dynamic residual approach	1 mode	2.80	3.15	2.45	3.50	4.20	3.85	0.00
	2 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00
	3 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00
	6 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00
Modal property difference approach	1 mode	2.80	3.15	2.45	3.50	4.21	3.83	0.12
	2 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00
	3 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00
	6 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00

properties. The initial guesses of the stiffness parameters are all assigned to be 3.5×10^4 N/m, different from actual values in Table 1. Because no noise is present, the regularization parameter λ is set to zero and weighting factors w_i for all measured modes are set identically. For each model updating approach, the updating is performed assuming different numbers of measured modes are available (i.e. modes corresponding to the 1, 2, 3, or 6 lowest natural frequencies). The initial and updated parameter values are summarized in Table 2. Shown in the table, both model updating approaches can achieve accurate solutions when the data is noise-free (the average errors are all close to or equal to zero).

Using noisy modal properties generated from

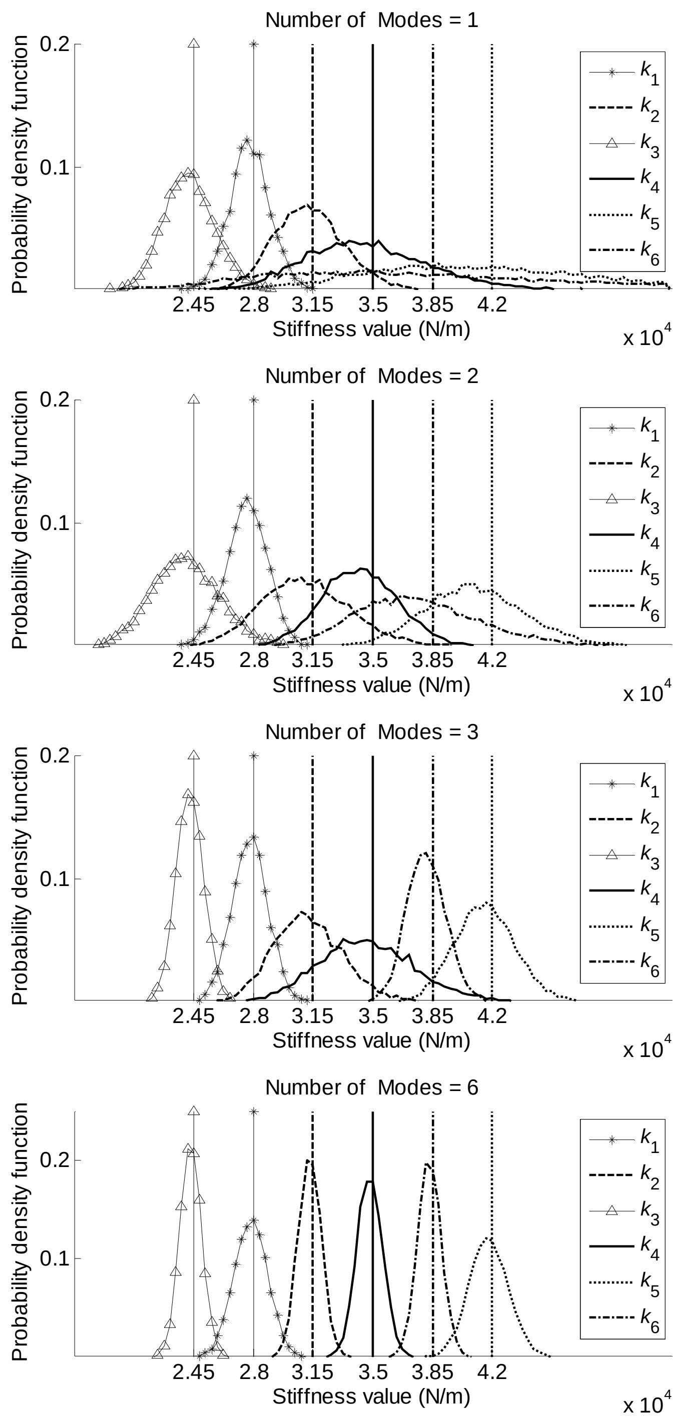

Figure 2. Case 1 – Probability density functions of updated parameters using the modal dynamic residual approach

Monte Carlo simulation, both modal property difference approach and modal dynamic residual approach are performed for model updating. Figure 2 shows the probability density functions of updated results through modal dynamic residual approach, when different numbers of modes are available. Figure 3 shows the results for modal property difference approach. The actual values of the updating parameters are marked at horizontal axis and represented using vertical lines in each plot. When the number of available modes increases for both updating approaches, the variance of some updated parameters decreases significantly (e.g. k_4 and k_5), and the bias from actual values also reduces. Both figures also show that in this example, the probability density function of each updated parameter is close to a normal distribution.

Table 3 summarizes the RMS error of each updated parameter, as well as the average RMS error among all the parameters. It can be concluded that when only the first mode is available, neither of the two approaches gives reliable results. As expected, updating results improve as the number of measured modes increases. When the number of available modes increases from 1 to 6, the average error decreases monotonically from 13.4% to 2.49% for modal dynamic residual approach, and from 10.9% to 4.49% for modal property difference approach. In this example, the modal dynamic residual approach performs better when more modes are available.

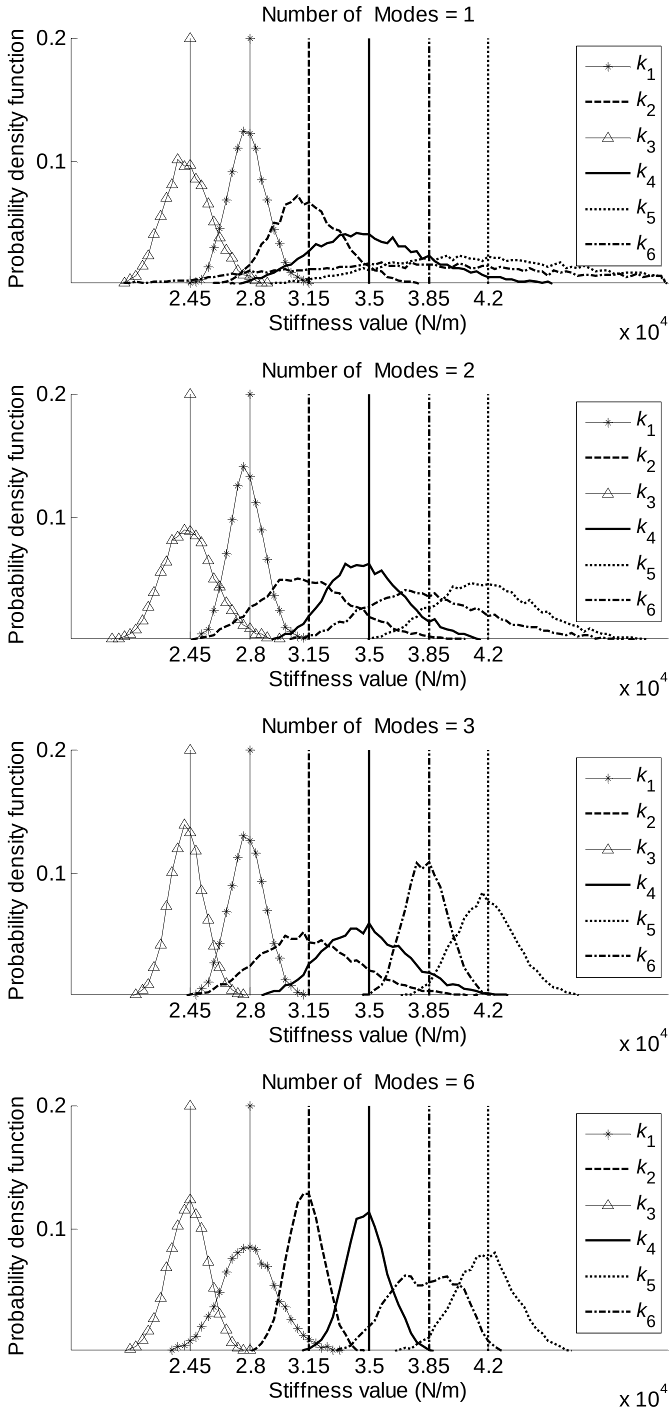

Figure 3. Case 1 – Probability density functions of updated parameters using the modal property difference approach

Table 3. RMS error of model updating results

Updating parameter		k_1 (%)	k_2 (%)	k_3 (%)	k_4 (%)	k_5 (%)	k_6 (%)	Avg. error (%)
Modal dynamic residual approach	1 mode	4.35	6.70	6.28	10.9	19.6	32.3	13.4
	2 modes	4.40	8.34	8.14	6.55	7.35	9.90	7.45
	3 modes	3.74	6.45	3.31	8.14	4.25	3.00	4.81
	6 modes	3.53	2.15	2.60	2.15	2.72	1.79	2.49
Modal property difference approach	1 mode	4.11	6.65	5.88	10.6	16.3	22.1	10.9
	2 modes	3.67	9.17	6.47	6.50	7.47	10.30	7.26
	3 modes	3.84	9.65	4.16	7.50	4.25	3.24	5.44
	6 modes	5.99	3.39	4.70	3.57	4.23	5.04	4.49

3.2 *Case 2: incomplete measurement*

In this case, only half of the DOFs are measured (Figure 4). Same as Case 1, noise-free scenario is studied first as the reference, where the regularization parameter λ is set to zero and weighting factors w_i for all measured modes are set identically. The initial and updated parameter values for both updating approaches are summarized in Table 4. The table shows that when only one mode is available, neither approach can accurately update the parameters.

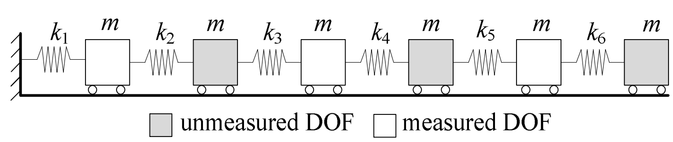

Figure 4. Measurement configuration

Table 4. Model updating results

Updating parameter		k_1	k_2	k_3	k_4	k_5	k_6	Avg. error (%)
		(10^4N/m)						
Initial Value		3.50	3.50	3.50	3.50	3.50	3.50	17.45
Modal dynamic residual approach	1 mode	2.82	2.73	2.86	3.50	4.19	4.20	6.69
	2 modes	2.81	3.21	2.43	3.6	4.08	3.78	1.77
	3 modes	2.81	3.22	2.46	3.54	4.15	3.85	0.89
	6 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00
Modal property difference approach	1 mode	2.82	2.70	2.89	3.78	3.73	3.48	10.29
	2 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00
	3 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00
	6 modes	2.80	3.15	2.45	3.50	4.20	3.85	0.00

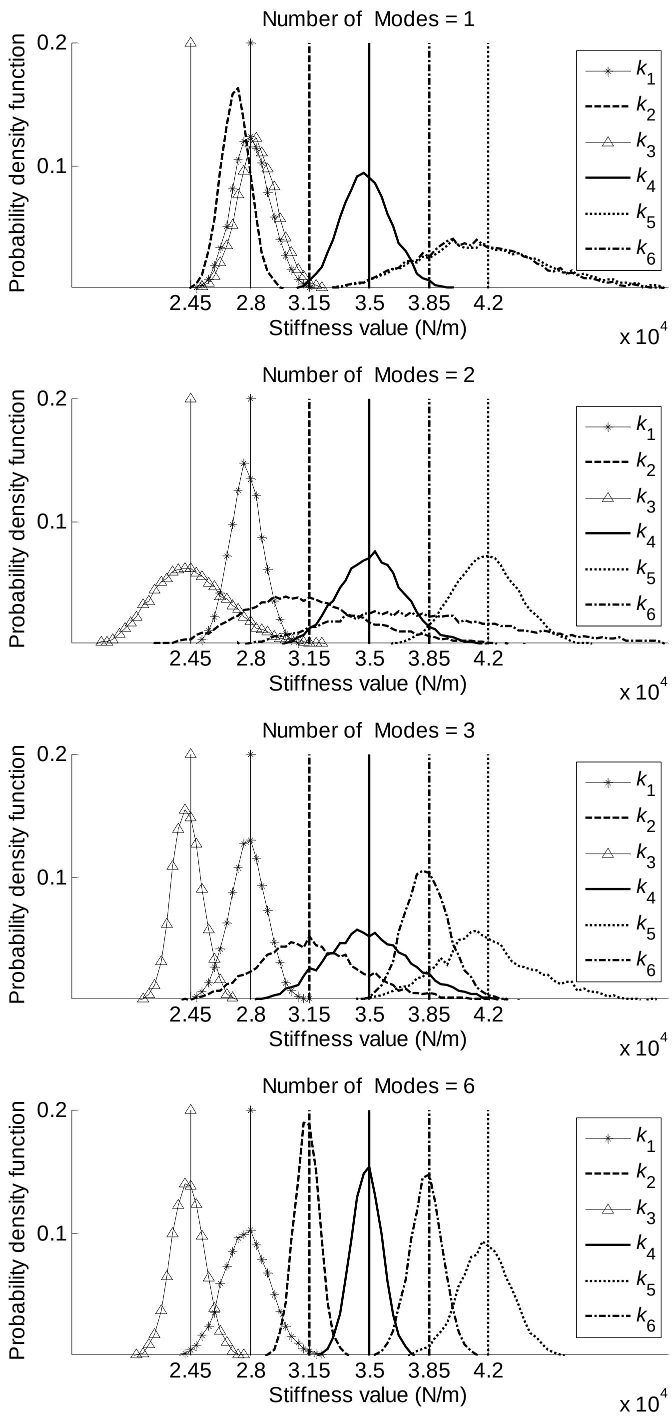

Figure 5. Case 2 – Probability density functions of updated parameters using the modal dynamic residual approach

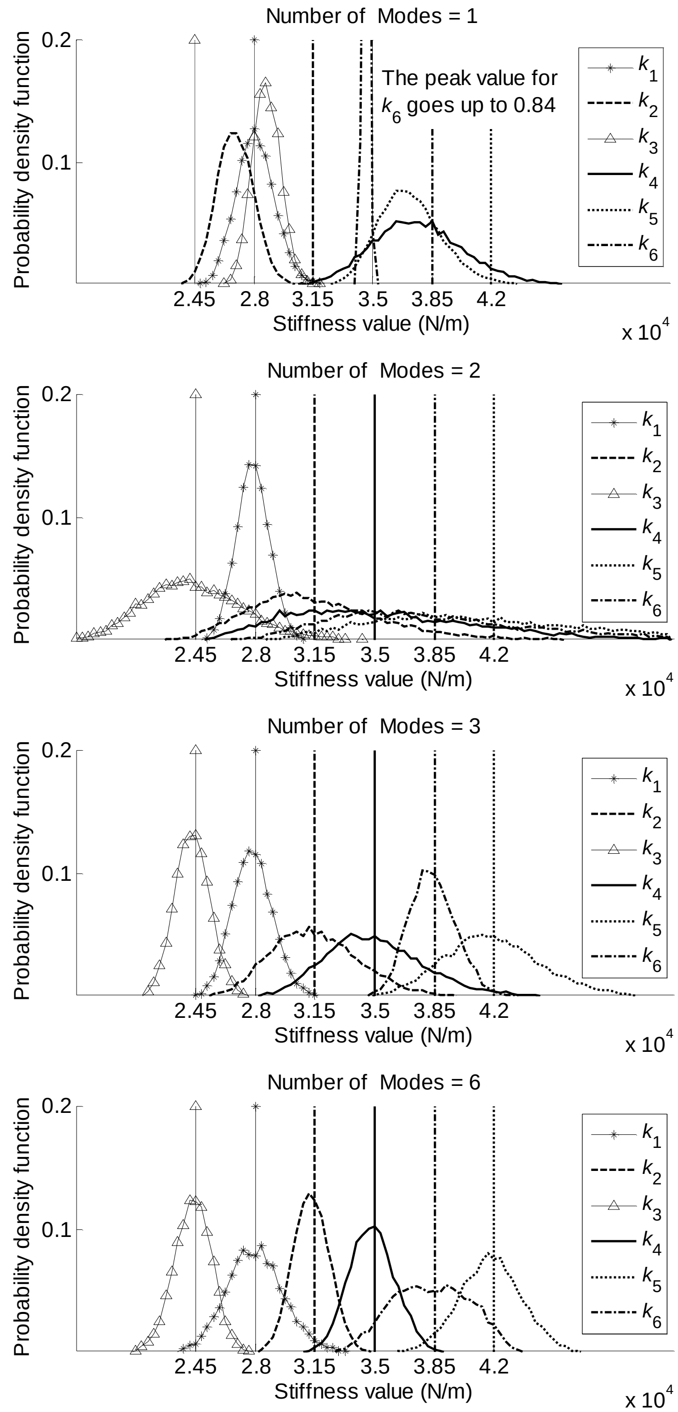

Figure 6. Case 2 – Probability density functions of updated parameters using the modal property difference approach

When more modes are available, both model updating approaches can achieve accurate solutions in the noise-free scenario (the average errors are all close to or equal to zero).

Similar to Case 1, both modal property difference and modal dynamic residual approaches are performed using noisy modal properties generated from Monte Carlo simulation. Figure 5 shows the probability density functions of updated results through modal dynamic residual approach, when different numbers of modes are available. Figure 6 shows the results for modal property difference approach. The actual values of the updating parameters are represented using vertical lines in each plot. When only one mode is available, most of the updated parameters are much biased from the actual parameter

Table 5. RMS error of model updating results

Updating parameter		k_1 (%)	k_2 (%)	k_3 (%)	k_4 (%)	k_5 (%)	k_6 (%)	Avg. error (%)
Modal dynamic residual approach	1 mode	4.16	13.57	17.54	4.33	10.04	13.20	10.47
	2 modes	3.54	12.28	9.64	5.84	4.68	15.28	8.54
	3 modes	3.93	10.22	3.60	7.60	7.28	3.51	6.02
	6 modes	4.96	2.29	3.99	2.58	3.67	2.51	3.33
Modal property difference approach	1 mode	4.20	14.4	18.4	11.4	11.8	9.66	11.6
	2 modes	3.47	14.8	12.6	18.7	21.7	25.4	16.1
	3 modes	4.20	8.47	4.20	8.65	7.01	3.60	6.02
	6 modes	6.06	3.39	4.53	3.85	4.34	5.78	4.66

values, similar to the observation from the average RMS errors for the noise-free scenario (Table 4). For example, when only 1 mode is available for the modal property difference approach, the estimation for parameter k_6 has a small variance but a large bias from actual parameter value of 3.85×10^4 N/m. When 2 or more modes are available, updating bias significantly reduces, and the variance of some updated parameters decreases considerably. In addition, similar as Case 1, Figure 5 and Figure 6 show that the probability density function of each updated parameter is close to a normal distribution in this example.

Table 5 summarizes the RMS error of each updated parameter, as well as the average RMS error of all the parameters using different numbers of available modes. Similar observation as in Case 1 can be made. For both approaches, the updating performance improves when the number of measured modes increases. Based on the average RMS error, the modal dynamic residual approach gives slightly better performance in general. The average RMS error of modal dynamic residual approach is lower than those of modal property difference approach for 1,2, or 6 modes. With 3 modes available, the average RMS errors are the closest between the two approaches. In addition, the average RMS error of modal dynamic residual approach decreases monotonically when the number of available modes increases.

4 CONCLUSIONS

This research investigates the robustness of two model updating approaches against measurement noise. The modal property difference approach minimizes the difference between experimental and simulated natural frequencies and mode shapes. The modal dynamic residual approach minimizes the modal dynamic residual of the generalized eigenvalue equation in structure dynamics. To improve performance of both approaches, weighting factors are applied to contributions from different modes in the optimization objective function. In the modal dynamic residual approach, a regularized objective function is found to improve updating performance.

Numerical simulation is performed using a 6-DOF spring-mass model to compare the performance of the two updating approaches. Two measurement cases are studied, one case with complete measurements at all DOFs and the other one with partial measurements at some DOFs. Monte Carlo simulation is conducted to generate experimental modal properties contaminated with noise. The probability density functions of the updated parameters appear to be close to normal distributions.

In addition, when only one mode is available, neither of the two model updating approaches provides satisfactory results. When more modes are available, the updating performance improves. This example also shows that the modal dynamic residual approach overall gives better results than the modal property difference approach. When more high-frequency modes are available for model updating, the average RMS errors using the dynamic residual approach are generally smaller than or almost equivalent to the errors using property difference approach. Furthermore, the average RMS error from the dynamic residual approach has a favorable trend of monotonically decreasing when number of available modes increases. In the future, more extensive analytical and numerical studies are needed on the convergence, accuracy, and computational efficiency of both model updating approaches under noisy measurements.

5 ACKNOWLEDGEMENT

This research is partially sponsored by the National Science Foundation, under grant numbers CMMI-1150700 and CMMI-1041607. The authors gratefully acknowledge the support. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

6 REFERENCES

Abdalla, M. O., Grigoriadis, K. M. & Zimmerman, D. C. (2000). Structural damage detection using linear matrix inequality methods. *Journal of Vibration and Acoustics*, 122(4), 448-455.

Ahmadian, H., Mottershead, J. E. & Friswell, M. I. (1998). Regularisation methods for finite element model updating. *Mechanical Systems and Signal Processing*, 12(1), 47-64.

Farhat, C. & Hemez, F. M. (1993). Updating finite element dynamic models using an element-by-element sensitivity methodology. *AIAA Journal*, 31(9), 1702-1711.

Friswell, M. I. & Mottershead, J. E. (1995). *Finite element model updating in structural dynamics*, Kluwer Academic Publishers, Dordrecht; Boston.

Hua, X. G., Ni, Y. Q., Chen, Z. Q. & He, X. H. (2012). Monte Carlo study of the effect of measurement noise in nodel updating with regularization. *Journal of Engineering Mechanics-Asce*, 138(1), 71-81.

Jaishi, B. & Ren, W. X. (2006). Damage detection by finite element model updating using modal flexibility residual. *Journal of Sound and Vibration*, 290(1-2), 369-387.

MathWorks Inc. (2005). *Control System Toolbox : for Uses with MATLAB® : Getting Started*, Version 6., MathWorks Inc., Natick, MA.

Moré, J. (1978). The Levenberg-Marquardt algorithm: Implementation and theory. *Numerical Analysis*, G. A. Watson, ed., Springer Berlin Heidelberg, 105-116.

Teughels, A., Maeck, J. & De Roeck, G. (2002). Damage assessment by FE model updating using damage functions. *Computers & Structures*, 80(25), 1869-1879.