An Iterative Convex Optimization Procedure for Structural System Identification

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ABSTRACT

Owing to the complexity of civil structures, structural behavior predicted by finite element models (built according to design drawings) is usually different from behavior of an actual structure in the field. To improve the prediction accuracy, finite element (FE) model updating can be performed based on sensor measurement from the actual structure. Numerous FE model updating algorithms have been developed in the past few decades. However, most existing algorithms suffer computational challenges when applied to large structures. The challenges are usually because that most existing algorithms attempt to solve non-convex optimization problems. As a result, the optimization process encounters convergence difficulty and cannot guarantee global optimum. To address the issue, this paper proposes an iterative convex optimization algorithm for FE model updating. The convex attribute of the optimization problem makes the solution process tractable and efficient. To validate the proposed algorithm, numerical simulation is conducted with a plane truss structure. The proposed approach successfully updates the simulation model and shows certain advantage over a conventional FE model updating algorithm.

INTRODUCTION

In modern structural analysis, extensive efforts have been devoted to developing accurate finite element (FE) models. However, simulation results from an FE model often differ from field testing results, which may be caused by various inaccuracies in the model. For example, in actual structures, support conditions are far more complicated than ideal hinges, fixed ends, or rollers commonly used in modeling. Besides, most structural components may deteriorate over time. An FE model based on original structural properties does not accurately reflect the deteriorated structure. To obtain a more reliable model, experimental data collected from the actual structure in the field can be used to update the model, which is known as FE model updating. The updated model is expected to predict structural response with higher fidelity. Furthermore, by tracking major property changes at individual structural components, model updating can assist in diagnosing structural damage.

During past few decades, various FE model updating methods have been developed and practically applied (Friswell and Mottershead 1995). A large amount of these methods utilize modal analysis results from field testing. Selected structural parameters are updated by forming an optimization objective function that minimizes the difference between experimental and simulated modal properties. In particular, early researchers started by minimizing difference between measured and simulated natural frequencies. For example, Zhang *et al.* (2000) proposed an eigenvalue sensitivity-based model updating approach that was applied on a scaled suspension bridge model. Salawu (1997) reviewed various model updating algorithms using natural frequencies, and concluded that changes in frequencies may not be sufficient enough for identifying system parameters. Therefore, other modal characteristics, e.g. mode shapes or modal flexibility, were investigated for model updating. For example, Moller and Friberg (1998) adopted the modal assurance criterion (MAC)-related function for updating the model of an industrial structure. FE model updating using changes in mode shapes and frequencies was applied for damage assessment of a reinforced concrete beam (Teughels *et al.* 2002). More recently, Yuen (2012) developed an efficient model updating algorithm using frequencies and mode shapes at only some selected degrees-of-freedom (DOFs) for a few modes. Jaishi and Ren (2006) proposed an objective function consisting of changes in frequencies, MAC related functions and modal flexibility for updating the model of a beam structure. Overall, most previous optimization objective functions that describe the difference in experimental and simulated modal properties are highly nonlinear and non-convex. As a result, conventional modal-based approaches suffer convergence issues, and may not guarantee global optimum.

To overcome the difficulty, this research investigates efficient model updating by formulating a convex optimization problem. The convex attribute guarantees global optimality of a solution, and makes the optimization process tractable and highly efficient (Boyd and Vandenberghe 2004; Grant and Boyd 2013; Lin *et al.* 2010). In addition, some DOFs are difficult to measure during field experiments. A modal expansion process using measurement at a limited number of DOFs is adopted in order to obtain experimental mode shapes for all DOFs. Due to modal expansion, formulation of the convex optimization problem is based upon an initial FE model. Therefore, an iterative convex optimization procedure is proposed for higher updating accuracy. After an updated model is obtained as the solution of convex optimization, the updated model is used again as an initial model to repeat the updating process till convergence. The rest of the paper is organized as follows. The formulation of iterative convex optimization procedure is presented first. Numerical validation on a plane truss model is then described. Performance of the proposed approach is compared with a conventional model updating approach. Finally, a summary and discussion are provided.

ITERATIVE CONVEX OPTIMIZATION FOR MODEL UPDATING

For a linear structural system, the system stiffness and mass matrices can be updated as:

$$\mathbf{K} = \mathbf{K}_0 + \sum_{i=1}^{n_\alpha} \alpha_i \mathbf{K}_{0,i} \qquad \mathbf{M} = \mathbf{M}_0 + \sum_{j=1}^{n_\beta} \beta_j \mathbf{M}_{0,j} \tag{1}$$

where $\mathbf{K}_0$ and $\mathbf{M}_0$ are the initial stiffness and mass matrices estimated prior to model updating; α_i and β_j are system stiffness and mass parameters to be updated (such as elastic modulus and density of each element); n_α and n_β represent the total number of updating parameters; $\mathbf{K}_{0,i}$ and $\mathbf{M}_{0,j}$ are constant matrices that represent contributions corresponding to the updating parameters. From Eq. (1), the system matrices are affine functions of updating parameters $\boldsymbol{\alpha} = \left[\alpha_1, \alpha_2, \cdots, \alpha_{n_\alpha}\right]^{\mathrm{T}}$, and $\boldsymbol{\beta} = \left[\beta_1, \beta_2, \cdots, \beta_{n_\beta}\right]^{\mathrm{T}}$.

In this study, a convex optimization formulation is adopted for model updating with optimization variables $\mathbf{x} = \left[\boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{s}\right]^{\mathrm{T}}$:

$$\text{Minimize} \qquad \max\left(s_1, s_2, \cdots, s_m\right) \tag{2a}$$

$$\text{Subject to} \qquad \left\|\left(-\omega_j^2 \mathbf{M} + \mathbf{K}\right) \boldsymbol{\psi}_j\right\| - s_j \leq 0; \quad j = 1 \ldots m \tag{2b}$$

$$\boldsymbol{\alpha}_L \leq \boldsymbol{\alpha} \leq \boldsymbol{\alpha}_U \qquad \boldsymbol{\beta}_L \leq \boldsymbol{\beta} \leq \boldsymbol{\beta}_U \tag{2c}$$

where m denotes the number of measured modes obtained from experimental data; $\mathbf{s} = \left[s_1, s_2, \cdots, s_m\right]^{\mathrm{T}}$ includes slack variables limiting the 2-norm of eigenvalue expressions in Eq. (2b); $\boldsymbol{\alpha}_L$ and $\boldsymbol{\beta}_L$ denote the element-wise lower bounds for vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, respectively; $\boldsymbol{\alpha}_U$ and $\boldsymbol{\beta}_U$ denote the element-wise upper bounds for vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, respectively; ω_j and $\boldsymbol{\psi}_j$ represent the j-th frequency and mode shape, obtained from experimental data. Note that the sign "$\leq$" in Eq. (2c) is overloaded to represent element-wise inequality.

It can be proved that the optimization problem in Eq. (2) satisfies the definition of a convex optimization problem (Boyd and Vandenberghe 2004; Zhu and Wang 2012) by assuming that the experimental frequencies and mode shapes (ω_j and $\boldsymbol{\psi}_j$, j = 1,2,…m) are constant. However, in practice, usually not all DOFs can be measured by sensors. This indicates that only incomplete mode shapes can be directly obtained from experimental data. To solve this issue, modal expansion can be performed (Kidder 1973):

$$\boldsymbol{\psi}_{j,U} = (-\mathbf{D}_{UU}^{-1} \mathbf{D}_{UM}) \boldsymbol{\psi}_{j,M} \tag{3}$$

where subscripts M and U represent the measured and unmeasured DOFs, respectively; $\boldsymbol{\psi}_{j,M}$ and $\boldsymbol{\psi}_{j,U}$ represent the measured and unmeasured entries of the j-mode shape vector. The expansion matrix $(-\mathbf{D}_{UU}^{-1} \mathbf{D}_{UM})$ comes from the eigenvalue problem of the initial structural model:

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_{MM} & \mathbf{D}_{MU} \\ \mathbf{D}_{UM} & \mathbf{D}_{UU} \end{bmatrix} = -\omega_j^2 \mathbf{M}_0 + \mathbf{K}_0 \quad (4)$$

As described before, the model updating process is based upon an initial FE model. The updated model can be used as an initial model again to repeat the updating process for higher accuracy. The procedures of the iterative convex optimization process for model updating are summarized as follows.

(1) Formulate the initial FE model ($\mathbf{K}_0$ and $\mathbf{M}_0$), e.g. according to design drawings;

(2) Modal expansion using experimental mode shapes measured at a limited number of DOFs (Eq. (3));

(3) Solve convex optimization problem for $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\mathbf{s}$ (Eq. (2));

(4) Use optimal $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ to update $\mathbf{K}_0$ and $\mathbf{M}_0$, and return to Step (2), until the updated variables converge.

NUMERICAL EXAMPLE

To validate the proposed convex optimization approach for model updating, a plane truss model is simulated (Figure 1). The truss model has 14 nodes, and each node has two translational DOFs. Table 1 summarizes the structural properties of the model. Different elastic moduli are assigned to top chords, diagonal & vertical bars, and bottom chords. Non-ideal support conditions are considered in this example. Vertical and horizontal springs are allocated at the left support to simulate a non-ideal hinge, while a vertical spring is allocated at the right support to simulate a non-ideal roller. Figure 2 shows the first six natural frequencies and mode shapes of the structure. For simplicity, these modal properties are directly used as the experimental results for model updating.

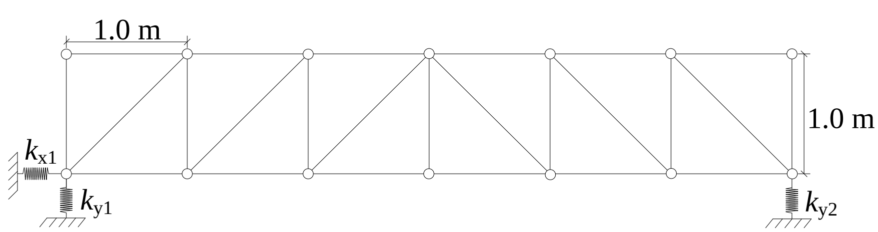

Figure 1. Plane Truss Structure

Table 1. Structural Properties

Property		Value
Elastic modulus (N/m^2)	Top chords	2.2×10^{11}
	Diagonal & vertical bars	2.1×10^{11}
	Bottom chords	1.9×10^{11}
Density (kg/m^3)		7,849
Cross-section area (m^2)		8×10^{-5}
Spring k_{y1} (N/m)		7×10^{6}
Spring k_{x1} (N/m)		2×10^{6}
Spring k_{y2} (N/m)		5×10^{6}

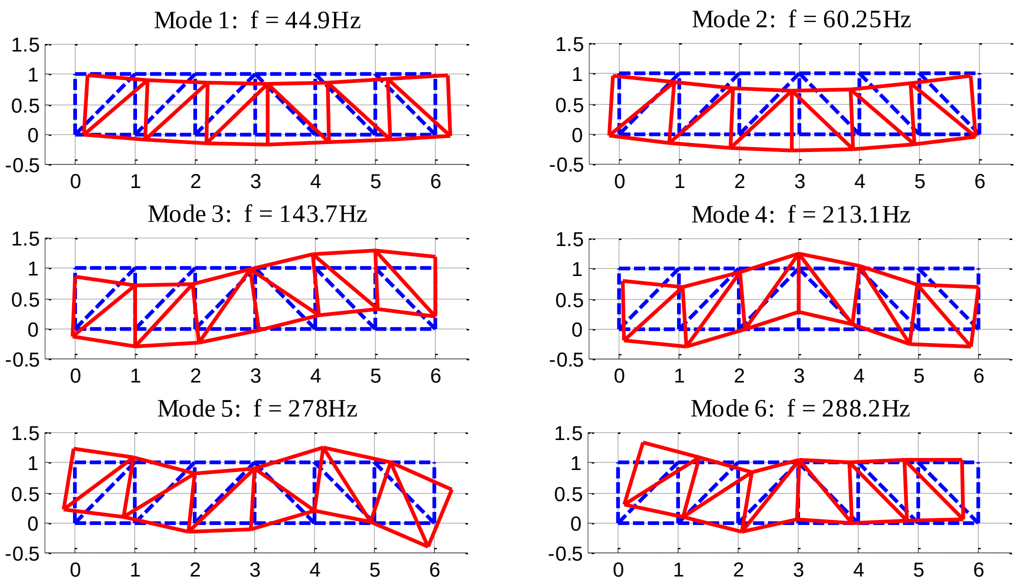

Figure 2. Modal Properties of the Plane Truss Structure

For comparison, a conventional model updating approach is also studied (Jaishi and Ren 2006). The conventional model updating formulation aims to minimize the difference between experimental and analytical natural frequencies and mode shapes. The optimization problem has optimization variables $\mathbf{x} = [\boldsymbol{\alpha}, \boldsymbol{\beta}]^{\mathrm{T}}$, which corresponds to stiffness and mass updating parameters.

$$\text{Minimize} \qquad \sum_{i=1}^{m}\left(\frac{f_{\mathrm{FE},i} - f_{\exp,i}}{f_{\exp,i}}\right)^2 + \sum_{i=1}^{m}\left(\frac{1-\sqrt{\mathrm{MAC}_i}}{\sqrt{\mathrm{MAC}_i}}\right)^2 \tag{5a}$$

$$\text{Subject to} \qquad \boldsymbol{\alpha}_L \le \boldsymbol{\alpha} \le \boldsymbol{\alpha}_U \qquad \boldsymbol{\beta}_L \le \boldsymbol{\beta} \le \boldsymbol{\beta}_U \tag{5b}$$

Here m denotes the number of measured modes; $f_{\mathrm{FE},i}$ and $f_{\exp,i}$ represent the analytical (from FE model) and experimental natural frequencies, respectively; MAC_i represents the modal assurance criterion evaluating the difference between the i-th analytical and experimental mode shapes.

A nonlinear least square optimization solver, 'lsqnonlin' in MATLAB toolbox (MathWorks Inc. 2005), is adopted to numerically solve the conventional model updating problem. The optimization solver seeks a minimum of the objective function in Eq. (5a) through Levenberg-Marquardt algorithm (Moré 1978), which uses a search direction interpolated between the Gauss-Newton direction and the steepest descent direction.

Two measurement cases are studied. Case 1 assumes all DOFs are measured. Because modal expansion is not needed, model updating through the proposed approach is achieved by solving the convex optimization problem (Eq. (2)) only once, i.e. without iteration. Case 2 assumes partial DOFs are measured. Because modal expansion is needed, the iterative convex optimization procedure is performed. For comparison, model updating of both cases are also performed using conventional formulation in Eq. (5). In this preliminary study, it is assumed to have perfect knowledge on structural mass. Therefore, only stiffness parameters $\boldsymbol{\alpha}$ are updated. After model updating, the root mean square (RMS) of the relative difference between optimal and actual parameters (Table 1) is calculated to evaluate the updating performance.

$$RMS = \sqrt{\frac{1}{n_\alpha}\sum_{i=1}^{n_\alpha}\left(\frac{p_{opt,i} - p_{act,i}}{p_{act,i}}\right)^2} \tag{6}$$

Here n_α denotes the number of stiffness parameters being updated; $p_{opt,i}$ and $p_{act,i}$ represent the optimal value after updating and the actual value, respectively.

Case 1 In this case, since all DOFs are measured, modal expansion is not necessary. Table 2 lists all the updating parameters, including elastic moduli and support spring stiffnesses. The initial values of the updating variables are randomly assigned to be different from actual values. Both the proposed approach and the conventional model updating approach are performed. For each approach, the updating is performed assuming different numbers of measured modes (i.e. modes corresponding to the 1, 2, or 6 lowest natural frequencies) are available. The initial and updated parameter values are summarized in Table 2.

Shown by Table 2, the conventional formulation (Eq. (5)) can achieve optimal solutions when two or more modes are available for model updating. However, when only one mode is available, results of the conventional formulation have 8.44% RMS error from actual values. The proposed convex optimization algorithm can always achieve optimal values, even when only one mode is available. The corresponding

Table 2. Model Updating Results using Complete Measurements

Updating parameter		Elastic modulus (10^{11} N/m^2) Top chords	Diag. & Vert.	Bottom chords	k_{y1} (10^6 N/m)	k_{y2} (10^6 N/m)	k_{x1} (10^6 N/m)	RMS error (%)
Initial value		2.000	2.000	2.000	6.000	6.000	6.000	82.4
Proposed approach	1 mode	2.200	2.100	1.900	7.000	5.000	2.000	0.00
	2 modes	2.200	2.100	1.900	7.000	5.000	2.000	0.00
	6 modes	2.200	2.100	1.900	7.000	5.000	2.000	0.00
Conventional approach	1 mode	2.318	2.373	1.810	6.612	5.659	2.033	8.44
	2 modes	2.200	2.100	1.900	7.000	5.000	2.000	0.00
	6 modes	2.200	2.100	1.900	7.000	5.000	2.000	0.00

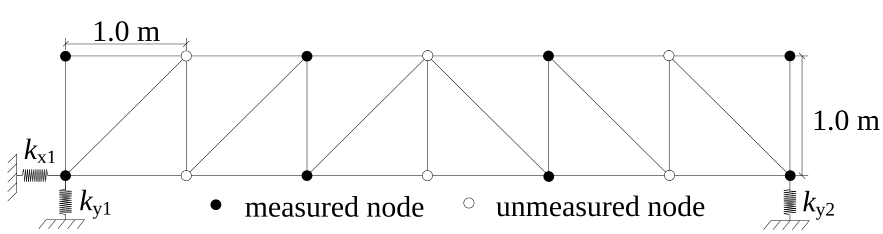

Figure 3. Measurement Configuration for Case 2

Table 3. Model Updating Results using Incomplete Measurements

Updating parameter		Elastic modulus (10^{11} N/m^2) Top chords	Elastic modulus (10^{11} N/m^2) Diag. & Vert.	Elastic modulus (10^{11} N/m^2) Bottom chords	k_{y1} (10^6 N/m)	k_{y2} (10^6 N/m)	k_{x1} (10^6 N/m)	RMS error (%)
Initial value		2.000	2.000	2.000	6.000	6.000	6.000	82.4
Proposed approach	1 mode	2.195	2.100	1.900	6.998	4.999	2.000	0.10
	2 modes	2.200	2.100	1.900	7.000	5.000	2.000	0.00
	6 modes	2.200	2.100	1.900	7.000	5.000	2.000	0.00
Conventional approach	1 mode	2.292	2.329	1.825	6.585	5.410	2.028	6.53
	2 modes	2.200	2.100	1.900	7.000	5.000	2.000	0.00
	6 modes	2.200	2.100	1.900	7.000	5.000	2.000	0.00

RMS errors are zero regardless how many modes are available.

Case 2 In this case, it is assumed only some nodes are measured by sensors. Modes shapes directly extracted from sensor data are only available at the measured DOFs. Figure 3 illustrates the measured nodes with solid dots. Both horizontal and vertical DOFs are measured at every solid dot. Similar to Case 1, both the conventional updating approach and the iterative convex optimization approach are applied. Table 3 summarizes the updating results of both approaches using different numbers of measured modes. It should be noted that the modal expansion (Eq. (3)) in the iterative convex method gives better estimation for modes at lower frequencies, but may introduce significant errors when expanding higher modes. Therefore, when multiple modes are available, it is decided to only use lower modes at the first few iterations for preliminary updating. Higher modes are then used to refine the updating results. The modal expansion process is not needed in conventional approach (Eq. (5)).

Table 3 demonstrates that the two updating approaches have similar performance as in previous Case 1. The conventional updating approach gives accurate solution, when two or more modes are available, but does not provide satisfactory results when only one mode is available. The iterative convex optimization approach works well when different numbers of modes are available. This advantage of the iterative convex optimization is promising, because in field testing, measurable structural vibration energy mainly occupies low frequency range. Modal properties of lower modes are easier to obtain reliably than higher modes.

CONCLUSION

This research investigates FE model updating through the formulation of a convex optimization problem. The convex attribute makes the solution process tractable and efficient. In order to apply the updating when only a limited number of DOFs are measured, modal expansion is performed to obtain the complete mode shapes. As required by modal expansion process, formulation of the convex optimization problem requires an initial FE model. An iterative optimization procedure is proposed to incorporate modal expansion at every iterative step, and to achieve higher updating accuracy. Numerical simulation is conducted to validate the proposed approach, and to compare the performance with a conventional approach minimizing the difference between analytical and experimental modal properties. It is shown that the proposed approach may give better performance when very limited numbers of modes are available. Nevertheless, more extensive analytical and numerical studies are needed on the convergence, accuracy, and computational efficiency of the proposed method.

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