

Decentralized \mathcal{H}_∞ Structural Control with Experimental Validation using Multi-subnet Wireless Sensing Feedback

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ABSTRACT

This study investigates the feasibility of deploying wireless communication and embedded computing technologies for structural control applications. A feedback control system involves a network of sensors and control devices. As control devices are becoming smaller, more cost effective and reliable, opportunities are now available to instrument a structure with large number of control devices. However, instrumenting a large scale centralized control system with cables can be time consuming and labor intensive. This study explores decentralized feedback control using wireless sensors incorporated with computational core and signal generation module. Decentralized control schemes are designed to make decisions based on data acquired from sensors located in the vicinity of a control device. Specifically, this paper describes an experimental study of a time-delayed decentralized structural control strategy that aims to minimize the \mathcal{H}_∞ norm of a closed-loop control system. The decentralized controller design employs a homotopy method that gradually transforms a centralized controller into multiple decentralized controllers. Linear matrix inequality constraints are included in the homotopic transformation to ensure optimal control performance. Different decentralized \mathcal{H}_∞ control architectures are implemented with a network of wireless sensing and control devices instrumented on a six-story scaled steel frame structure. The sensor network supports simultaneous communication within multiple wireless subnets. Experimental tests are conducted to demonstrate the performance of the wireless decentralized control schemes.

INTRODUCTION

Utilizing a network of sensors, controllers and control devices, feedback control systems can potentially mitigate excessive dynamic responses of a structure subjected to strong dynamic loads, such as earthquakes or typhoons (Housner *et al.* 1997; Spencer and Nagarajaiah 2003). For a large structure, the instrumentation of a cable-based communication network that connects large number of sensors, control devices, and controllers can be quite costly. Furthermore, maintaining the reliability and performance of a large-scale inter-connected real-time system can be challenging. This study

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investigates the feasibility of deploying wireless communication and embedded computing technologies for structural control applications.

As control devices are becoming smaller, more cost effective and reliable, opportunities are now available to instrument a structure with large number of control devices. Scalability of current sensing and control devices for large-scale control systems are hindered by their dependence on centralized control strategy where a central controller is responsible for acquiring data and making control decisions. To mitigate some of the difficulties with centralized feedback control systems, decentralized control strategies can be explored (Lunze 1992). Decentralized control schemes can be designed to make decisions based on data acquired from sensors located in the vicinity of a control device. Furthermore, decentralized feedback control can take advantage of wireless sensors incorporated with computational core and signal generation module (Wang et al. 2007). For a wireless sensing and control system, the wireless sensors can not only collect and communicate sensor data, but also make optimal control decisions and directly command control devices in real-time. With the wireless sensor devices, decentralized control algorithms can be embedded and performed in a parallel and distributive manner.

This paper describes an experimental study of a time-delayed decentralized structural control strategy that aims to minimize the \mathcal{H}_∞ norm of a closed-loop control system. \mathcal{H}_∞ control can offer excellent control performance particularly when “worst-case” external disturbances are encountered. Centralized \mathcal{H}_∞ controller design in the continuous-time domain for structural control has been studied by many researchers (Johnson *et al.* 1998; Lin *et al.* 2006; Yang *et al.* 2004). Their studies have shown the effectiveness of centralized \mathcal{H}_∞ control for civil structures. For example, it has been shown that \mathcal{H}_∞ control design can achieve excellent performance in attenuating transient vibrations of structures (Chase et al. 1996). We have also conducted numerical simulations to evaluate the performance of time-delayed decentralized \mathcal{H}_∞ control (Wang 2009; Wang *et al.* 2009). The decentralized controller design employs a homotopy method that gradually transforms a centralized controller into multiple decentralized controllers. Linear matrix inequality constraints are included in the homotopic transformation to ensure optimal control performance. This paper presents the results of an experimental study on the time-delayed decentralized \mathcal{H}_∞ controller design. Different decentralized \mathcal{H}_∞ control architectures are implemented with a network of wireless sensing and control devices instrumented on a six-story scaled steel frame structure. Shake table experiments are conducted to examine the performance of different decentralized control architectures.

The paper is organized as follows. First, the formulation for decentralized \mathcal{H}_∞ controller design is summarized. The experimental setup of the six-story steel frame structure instrumented with wireless sensing and control units is then described. Experimental results will then be presented to evaluate the effectiveness of the wireless decentralized \mathcal{H}_∞ control strategies. Finally, this paper is concluded with a brief summary and discussion.

FORMULATION FOR TIME-DELAYED DECENTRALIZED STRUCTURAL CONTROL

Fig. 1 depicts the schematics of a structural control system defined for this study. For a structural model with n degrees-of-freedom (DOF) and instrumented with n_u control devices, the discrete-time system dynamics can be written as (Wang *et al.* 2009):

$$\begin{cases} \mathbf{x}_s[k+1] = \mathbf{A}_d \mathbf{x}_s[k] + \mathbf{E}_d \mathbf{w}_1[k] + \mathbf{B}_d \mathbf{u}[k] \\ \mathbf{z}[k] = \mathbf{C}_z \mathbf{x}_s[k] + \mathbf{F}_z \mathbf{w}_1[k] + \mathbf{D}_z \mathbf{u}[k] \\ \mathbf{m}[k] = \mathbf{C}_m \mathbf{x}_s[k] + \mathbf{F}_m \mathbf{w}_1[k] + \mathbf{D}_m \mathbf{u}[k] \end{cases} \quad (1)$$

In Eq. (1), $\mathbf{x}_s[k] \in \mathbb{R}^{2n \times 1}$, $\mathbf{w}_1[k] \in \mathbb{R}^{n_u \times 1}$ and $\mathbf{u}[k] \in \mathbb{R}^{n_u \times 1}$ denote the state vector, the external excitation vector, and control force vector, respectively. Assume a lumped mass structural model with n floors,

the state vector, $\mathbf{x}_s = [q_1 \quad \dot{q}_1 \quad q_2 \quad \dot{q}_2 \quad \dots \quad q_n \quad \dot{q}_n]^T$, consists of the relative displacement q_i and relative velocity \dot{q}_i (with respect to the ground) for each floor i , $i = 1, \dots, n$. The matrices $\mathbf{A}_d \in \mathbb{R}^{2n \times 2n}$, $\mathbf{E}_d \in \mathbb{R}^{2n \times n_{w_1}}$, and $\mathbf{B}_d \in \mathbb{R}^{2n \times n_u}$ are, respectively, the discrete-time dynamics, excitation influence, and control influence matrices. The vectors, $\mathbf{z}[k] \in \mathbb{R}^{n_z \times 1}$ and $\mathbf{m}[k] \in \mathbb{R}^{n_m \times 1}$ represent, respectively, the response output (to be controlled using the feedback loop) and the sensor measurement vector. Correspondingly, the matrices \mathbf{C}_z , \mathbf{F}_z , and \mathbf{D}_z are termed the output parameter matrices, and the matrices \mathbf{C}_m , \mathbf{F}_m , and \mathbf{D}_m are the measurement parameter matrices.

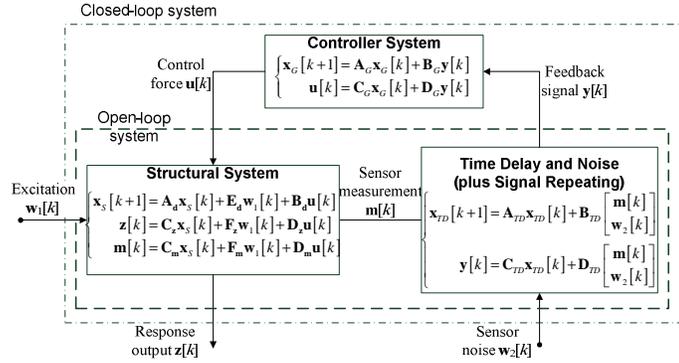


Fig. 1. Schematics of a control system

Suppose time delay over a sampling period ΔT exists for the sensor measurement signal $\mathbf{m}[k]$, for example, due to computational and/or communication overhead. Taking into consideration of sensor noise, denoted as $\mathbf{w}_2[k] \in \mathbb{R}^{n_{w_2} \times 1}$, the time-delay discrete system can be defined as:

$$\begin{cases} \mathbf{x}_{TD}[k+1] = \mathbf{A}_{TD} \mathbf{x}_{TD}[k] + \mathbf{B}_{TD} \begin{bmatrix} \mathbf{m}[k] \\ \mathbf{w}_2[k] \end{bmatrix} \\ \mathbf{y}[k] = \mathbf{C}_{TD} \mathbf{x}_{TD}[k] + \mathbf{D}_{TD} \begin{bmatrix} \mathbf{m}[k] \\ \mathbf{w}_2[k] \end{bmatrix} \end{cases} \quad (2)$$

Here, we let $\mathbf{A}_{TD} = \mathbf{0}$, $\mathbf{B}_{TD} = [\mathbf{I} \quad \mathbf{0}]$, $\mathbf{C}_{TD} = \mathbf{I}$, and $\mathbf{D}_{TD} = [\mathbf{0} \quad s_{w_2} \mathbf{I}]$. The inputs to the time-delay system are the measurement signal $\mathbf{m}[k]$ and the sensor noise $\mathbf{w}_2[k]$. The output of the time-delay system is the delayed noisy signal $\mathbf{y}[k]$, which is the feedback signal to be used for control decisions. The parameter s_{w_2} is a scaling factor representing sensor noise level. For simplicity, we assume the same scaling factor applies to all sensors, although, in general, different scaling factors can be assigned to different sensors by modifying the diagonal entries in the matrix $s_{w_2} \mathbf{I}$. The formulation can also be extended to model multiple time delay steps as well as different time delays for different channels.

The dynamical system as described by Eqs. (1) and (2) constitutes an open-loop system (Fig. 1). The sensor measurement vector $\mathbf{m}[k]$ from the structural system becomes the input to the time-delay system. The inputs of the open-loop system include the excitation $\mathbf{w}_1[k]$, the sensor noises $\mathbf{w}_2[k]$, and the control forces $\mathbf{u}[k]$ while the outputs in the open-loop system is thus equal to the total number of state variables in the structural system and the time-delay system, *i.e.* $n_{OL} = 2n + n_{TD}$. The structural system and the time-delay system can be cascaded (for example, using the `sysic` command in the Matlab Robust Control Toolbox (Chiang and Safonov 1998)), in the following form:

$$\begin{cases} \mathbf{x}[k+1] = \mathbf{A} \mathbf{x}[k] + \mathbf{B}_1 \mathbf{w}[k] + \mathbf{B}_2 \mathbf{u}[k] \\ \mathbf{z}[k] = \mathbf{C}_1 \mathbf{x}[k] + \mathbf{D}_{11} \mathbf{w}[k] + \mathbf{D}_{12} \mathbf{u}[k] \\ \mathbf{y}[k] = \mathbf{C}_2 \mathbf{x}[k] + \mathbf{D}_{21} \mathbf{w}[k] + \mathbf{D}_{22} \mathbf{u}[k] \end{cases} \quad (3)$$

where $\mathbf{w} = [\mathbf{w}_1^T \quad \mathbf{w}_2^T]^T \in \mathbb{R}^{n_w \times 1}$ contains both the external excitation \mathbf{w}_1 and the sensor noise \mathbf{w}_2 .

For feedback control, the controller system takes the signal $\mathbf{y}[k]$ as input and outputs the desired (optimal) control force vector $\mathbf{u}[k]$ according to the following state-space equations:

$$\begin{cases} \mathbf{x}_G[k+1] = \mathbf{A}_G \mathbf{x}_G[k] + \mathbf{B}_G \mathbf{y}[k] \\ \mathbf{u}[k] = \mathbf{C}_G \mathbf{x}_G[k] + \mathbf{D}_G \mathbf{y}[k] \end{cases} \quad (4)$$

where \mathbf{A}_G , \mathbf{B}_G , \mathbf{C}_G and \mathbf{D}_G are the parametric matrices of the controller to be computed and, for convenience, are often collectively denoted by a controller matrix $\mathbf{G} \in \mathbb{R}^{(n_G+n_u) \times (n_G+n_y)}$ as:

$$\mathbf{G} = \begin{bmatrix} \mathbf{A}_G & \mathbf{B}_G \\ \mathbf{C}_G & \mathbf{D}_G \end{bmatrix} \quad (5)$$

In this study, we assume both the controller and the open-loop system have the same number of states, i.e. $\mathbf{A}_G \in \mathbb{R}^{n_G \times n_G}$ and $n_G = n_{OL}$.

A decentralized control strategy can be defined by specifying a sparsity pattern in the controller matrices \mathbf{A}_G , \mathbf{B}_G , \mathbf{C}_G and \mathbf{D}_G . The feedback signals $\mathbf{y}[k]$ and the control forces $\mathbf{u}[k]$ are divided into N groups. For each group of control forces, the feedback signals corresponding to the communication patterns are grouped accordingly to reflect the decentralized control decisions. The decentralized architecture, denoted by a set of controllers \mathbf{G}_I , \mathbf{G}_{II} , ..., and \mathbf{G}_N , can be obtained by rewriting the controller matrices in block diagonal forms:

$$\begin{aligned} \mathbf{A}_G &= \text{diag}(\mathbf{A}_{G_I}, \mathbf{A}_{G_{II}}, \dots, \mathbf{A}_{G_N}) \\ \mathbf{B}_G &= \text{diag}(\mathbf{B}_{G_I}, \mathbf{B}_{G_{II}}, \dots, \mathbf{B}_{G_N}) \\ \mathbf{C}_G &= \text{diag}(\mathbf{C}_{G_I}, \mathbf{C}_{G_{II}}, \dots, \mathbf{C}_{G_N}) \\ \mathbf{D}_G &= \text{diag}(\mathbf{D}_{G_I}, \mathbf{D}_{G_{II}}, \dots, \mathbf{D}_{G_N}) \end{aligned} \quad (6)$$

The control system in Eq. (4) is thus equivalent to a set of uncoupled decentralized control subsystems, each requiring only one group of feedback signals to determine the desired (optimal) control forces for that subsystem.

$$\begin{cases} \mathbf{x}_{G_I}[k+1] = \mathbf{A}_{G_I} \mathbf{x}_{G_I}[k] + \mathbf{B}_{G_I} \mathbf{y}_I[k] \\ \mathbf{u}_I[k] = \mathbf{C}_{G_I} \mathbf{x}_{G_I}[k] + \mathbf{D}_{G_I} \mathbf{y}_I[k] \end{cases}, \dots, \quad (7)$$

$$\begin{cases} \mathbf{x}_{G_N}[k+1] = \mathbf{A}_{G_N} \mathbf{x}_{G_N}[k] + \mathbf{B}_{G_N} \mathbf{y}_N[k] \\ \mathbf{u}_N[k] = \mathbf{C}_{G_N} \mathbf{x}_{G_N}[k] + \mathbf{D}_{G_N} \mathbf{y}_N[k] \end{cases}$$

A decentralized \mathcal{H}_∞ structural control design using a homotopy method with linear matrix inequality (LMI) constraints is adopted in this study (Wang 2009). Let $\mathbf{H}_{zw} \in \mathbb{C}^{n_z \times n_w}$ represent the discrete-time closed-loop transfer function from the disturbance \mathbf{w} to the output response \mathbf{z} . The objective of \mathcal{H}_∞ control is to minimize the \mathcal{H}_∞ -norm of the closed-loop system:

$$\|\mathbf{H}_{zw}\|_\infty = \sup_{\omega \in [-\omega_N, \omega_N]} \bar{\sigma}[\mathbf{H}_{zw}(e^{j\omega\Delta T})] \quad (8)$$

where ω represents angular frequency, $\omega_N = \pi/\Delta T$ is the Nyquist frequency, j is the imaginary unit, and $\bar{\sigma}[\cdot]$ denotes the largest singular value of a matrix. In essence, the \mathcal{H}_∞ -norm represents the

largest amplification gain from the disturbance \mathbf{w} to the output \mathbf{z} within the Nyquist frequency range. The homotopy method described by Zhai, *et al.* (2001) for designing decentralized \mathcal{H}_∞ controllers in continuous-time domain is adapted for the discrete-time feedback control problem. The method gradually transforms a centralized controller into a set of uncoupled decentralized controllers corresponding to the decentralized feedback patterns. For each homotopy step, LMI constraints are used to ensure the closed-loop \mathcal{H}_∞ norm performance.

SHAKE-TABLE EXPERIMENTS WITH A SIX-STORY LABORATORY STRUCTURE

To study the performance of the decentralized \mathcal{H}_∞ structural control scheme with wireless sensing and actuation devices, shake table experimental tests on a six-story scaled structure were conducted at the National Center for Research on Earthquake Engineering (NCREE) in Taipei, Taiwan. This section describes the experimental setup, control formulation, and test results.

Experimental Setup for Decentralized Wireless Feedback Control

An experimental six-story steel frame structure, instrumented with RD-1005-3 magnetorheological (MR) dampers manufactured by Lord Corporation, is designed and constructed by researchers at NCREE (see Fig.2a). The structure is mounted on a $5\text{m} \times 5\text{m}$ six-DOF shake table, which can generate ground excitations with frequencies spanning from 0.1Hz to 50Hz. For this study, only longitudinal excitations are used. Accelerometers, velocity meters, and linear variable displacement transducers (LVDT) are instrumented on the shake table and on every floor to measure the dynamic responses of the structure. The sensors are interfaced to a high-precision wire-based data acquisition (DAQ) system at the NCREE facility; the DAQ system is set to operate with a sampling rate of 200 Hz.

The basic configuration of the prototype wireless sensing and control system is schematically shown in Fig.2b. A total of six Narada wireless units (Swartz and Lynch 2009) are installed in accordance with the deployment strategy. Each wireless unit consists of four functional modules: sensor signal digitization, computational core, wireless communication, and command signal generation. In the experiments, every wireless unit collects velocity data at its own floor and the floor above from the Tokyo Sokushin VSE15-D velocity meters that provide absolute velocity

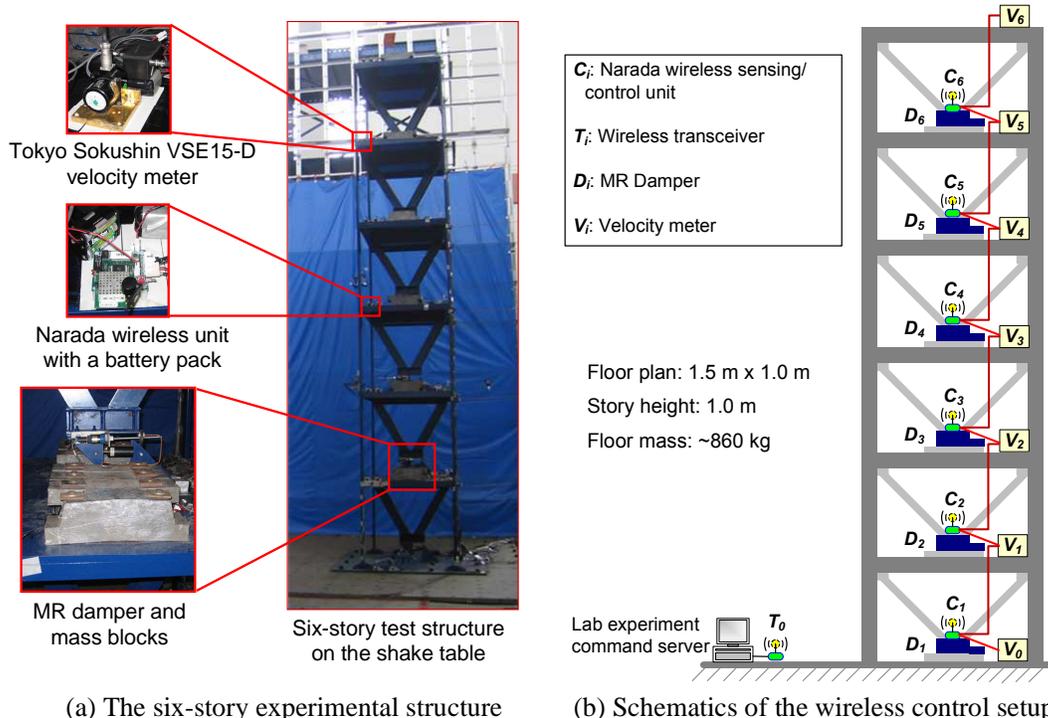


Fig.2. Experimental setup for \mathcal{H}_∞ feedback control using wireless sensing/control units

measurements. The sensitivity of the velocity meter is 10V/(m/s) with a measurement limit of ± 1 m/s. A remote data and command server with a wireless transceiver is also included for initiating each test run, and for retrieving test data from the six wireless units.

In addition to collecting and transmitting the velocity data, each wireless unit (C_1 through C_6 in Fig.2b) sends command signal to the MR damper that it is connected to. The damper on each floor is connected to the upper floor through a V-brace (Fig. 2a). Each damper can provide a maximum damping force over 2kN. The damping properties can be changed by the command voltage signal (ranging from 0 to 0.8V) through an input current source, which determines the electric current of the electromagnetic coil in the MR damper. The current then generates a variable magnetic field that sets the viscous damping properties of the MR damper. Calibration tests are first conducted on the MR dampers before mounting them onto the structure and a modified Bouc-Wen force-displacement model is developed for the damper (Lu *et al.* 2008). In the feedback control tests, the hysteresis model parameters for the MR dampers are an integral element of the control procedure embedded in the wireless units for calculating command voltages for the dampers.

Controller Design for the Six-story Experimental Structure

For the experimental setup, the velocity differences between every two neighboring floors are measured by the wireless units as the sensor measurements $\mathbf{m}[k]$ in Eq. (1):

$$\mathbf{m} = [\dot{q}_1 \quad \dot{q}_2 - \dot{q}_1 \quad \cdots \quad \dot{q}_n - \dot{q}_{n-1}]^T \quad (9)$$

The measurement matrices associated with the setup are therefore chosen as:

$$[\mathbf{C}_m]_{n \times 2n}(i, j) = \begin{cases} 1 & \text{if } j = 2i \\ -1 & \text{if } j = 2i - 2 \\ 0 & \text{otherwise} \end{cases}, \mathbf{F}_m = \mathbf{0}, \mathbf{D}_m = \mathbf{0} \quad (10)$$

When formulating the \mathcal{H}_∞ controllers, inter-story drifts are considered as the major controlling factors for minimizing the dynamic responses and the output matrices are defined as:

$$\mathbf{C}_z = \begin{bmatrix} \mathbf{C}_{z_1} \\ \mathbf{0}_{n \times 2n} \end{bmatrix} \text{ where } [\mathbf{C}_{z_1}]_{n \times 2n}(i, j) = \begin{cases} \sqrt{300} & \text{if } j = 2i - 1 \\ -\sqrt{300} & \text{if } j = 2i - 3 \\ 0 & \text{otherwise} \end{cases}, \mathbf{F}_z = \mathbf{0}, \mathbf{D}_z = \begin{bmatrix} \mathbf{0}_{n \times n} \\ \mathbf{I}_{n \times n} \end{bmatrix} \times 10^{-4.5} \quad (11)$$

Four decentralized/centralized feedback control architectures are adopted in the control experiments (Fig. 3). The degrees of centralization (DC) of different architectures reflect the different communication network configurations, with each wireless channel representing one communication subnet. The wireless units assigned to a subnet are allowed to access the wireless sensor data within that subnet. As an example, for case DC2, each wireless channel covers only two stories and a total of three wireless channels (subnets) are in operation. During the experiments, very little interference among different wireless channels has been observed even when the channels operate simultaneously. For case DC1, each wireless unit only utilizes the velocity difference between two neighboring floors for control decisions; therefore, no data transmission is required. For case DC4, one wireless channel (subnet) is used for all six wireless units, which is equivalent to a centralized feedback pattern.

The control sampling time step (or frequency) for each configuration is determined by the wireless communication and embedded computations. For the Narada units with Chipcon CC2420 radio, each wireless data transmission takes about 1.5ms to 2ms. For example, in case DC2, approximately 4ms are needed at every time step for the two wireless units in each channel to transmit the sensor data. The computational procedures performed by a wireless unit include calculating the desired control force for the MR damper (as described in Eq. (7)), updating the damper hysteresis model, and determining appropriate command signal for the damper. In this study, the computational time

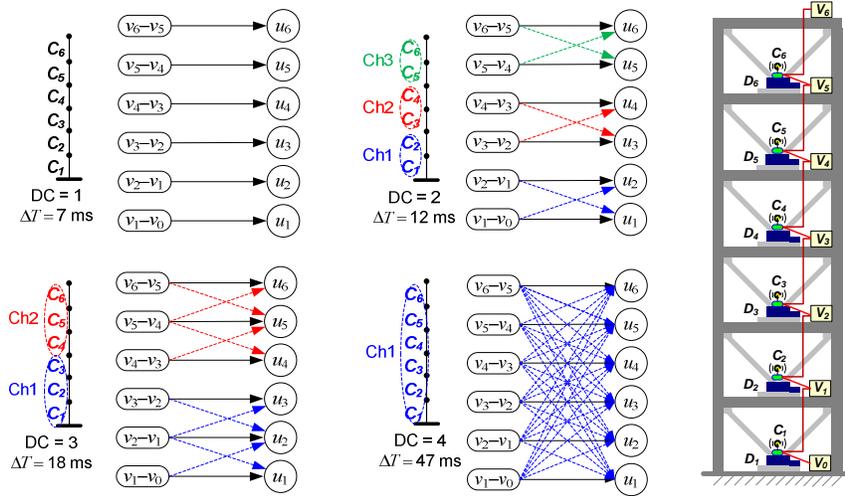


Fig. 3. Multiple feedback control architectures and the associated sampling time step lengths.

constitutes the dominant part of the feedback time delay, which, for case DC2, is set to be 12ms. Different decentralized architectures with different communication configurations require different computational demand on the wireless unit, which leads to different time delays as shown in Fig. 3.

Control Performance of Wireless Decentralized \mathcal{H}_∞ Feedback Control

The 1940 El Centro NS (Imperial Valley Irrigation District Station) earthquake excitation with the peak ground acceleration (PGA) scaled to 1m/s^2 is employed in this study. Fig. 4a shows the simulated peak inter-story drifts for different control architectures in Fig. 3, as well as two passive cases where the damper command voltages are fixed to the maximum (0.8V) and minimum (0V) values, respectively. Among all the passive and feedback control cases, the simulation results indicate that the feedback control case DC3 achieves the most uniform inter-story drifts among the six stories while DC4 shows smaller inter-story drifts on the upper floors.

Fig. 4b presents the experimental peak inter-story drifts for the two passive and the four feedback control cases. Although the peak drift values from the experiments are somewhat different from the simulated values, the results are of reasonable agreement. As shown in Fig. 4b, the \mathcal{H}_∞ feedback control through real-time wireless sensing feedback can achieve better performance than the passive control cases (with damper command voltages set to 0.8V or 0V). For this set of experiments, the centralized case DC4 achieves slightly better control performance than the decentralized case DC3. Such slight difference could well be attributed to the structure and damper model uncertainties that need further investigations. Both simulation and experimental results reveal the importance of a balance between minimizing time delays and the availability of sensor measurement data for optimal control decision.

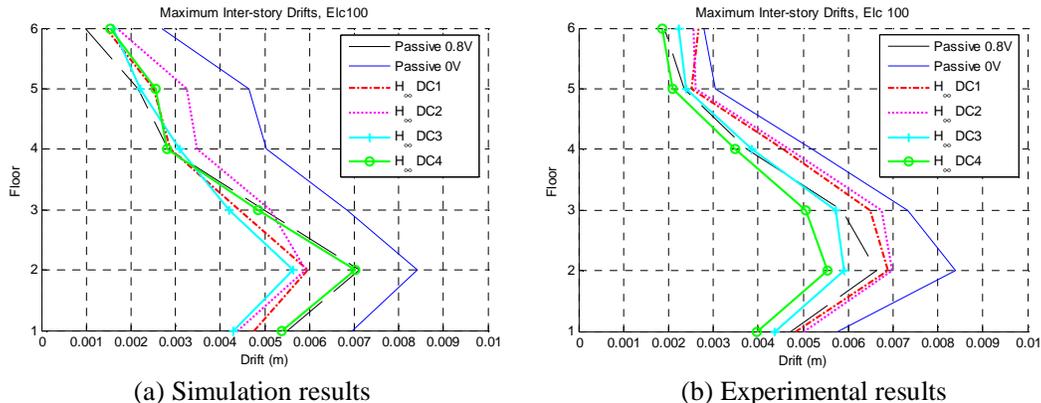


Fig. 4. Peak inter-story drifts for El Centro ground excitation with PGA scaled to 1m/s^2

SUMMARY AND DISCUSSION

This paper presents an experimental study for a time-delayed decentralized \mathcal{H}_∞ structural control design using homotopic transformation through linear matrix inequalities. A multi-channel network that allows simultaneous wireless communication has been successfully implemented for decentralized feedback control. Both simulation and experimental results demonstrate that the decentralized \mathcal{H}_∞ structural control approach using wireless sensing and embedded computing are viable and can outperform passive controls. Future work may explore the implementation of wireless sensor data feedback with information overlapping between multiple control groups, as well as the adoption of embedded hardware that minimize computational time delay.

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