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Implementation of Wireless Monitoring Systems for Modal Analysis of Bridges along a Korean Test Road

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Abstract: A wireless monitoring system is proposed for monitoring the behavior and integrity of highway bridges. At the core of the system is a low-cost, low-power wireless sensor node whose hardware design is optimized for structural monitoring applications. Furthermore, a rich set of computational tools can be embedded into the sensor's computational core to offer data processing capabilities at the sensor. The project is part of a US-Korea collaboration focused on the advancement of new sensing technologies for monitoring bridges. The performance and reliability of the proposed wireless monitoring system is validated on two medium-span bridges situated along an experimental test road in South Korea. The modal properties of each instrumented bridge are successfully extracted from output-only response data collected by the wireless monitoring system.

1. Introduction: The collapse of the I-35W Bridge (Minneapolis, Minnesota) on August 1, 2007 is a poignant reminder of what can happen when bridge inspection fails to identify distress and deterioration within a bridge. The failure of bridges in the United States is not as rare of an event as the public may believe. For example, between 1989 and 2000, a total of 134 bridges are reported to have partially or totally collapsed in the United States due to triggering events (*e.g.*, earthquake or vehicle collision), design and construction error, and undetected structural deterioration (*e.g.*, scour, fatigue) [1]. Many of the bridge management practices in use today have resulted from bridge failures. For example, the National Bridge Inspection Program (NBIP) was a direct result of public outcry after the Point Pleasant Bridge (crossing the Ohio River on the Ohio-West Virginia border) collapse in 1967. Today, the NBIP requires bi-annual visual inspection of every highway bridge in the United States to ensure public safety [2]. Bridge safety is an important societal issue for not only the United States, but for every developed nation in the world. For example, recent bridge failures (*e.g.,* New Haengju Bridge [3] and Seongsu Bridge [4]) have led to the implementation of more rigorous management procedures for bridges in Korea. Specifically, stringent visual inspection guidelines and the installation of permanent monitoring systems have both been implemented in Korea.

Today, visual inspection remains the predominant engineering tool used to assess the condition and safety of bridges. While visual inspection has been proven to be an effective approach, it suffers from some drawbacks. First, the approach is labor intensive since it requires a trained inspector to execute. Second, the approach is only capable of observing signs of distress from the surface of the structure. Deterioration hidden below the surface of the structure (*e.g.,* reinforcement corrosion) could go undetected. Third, the approach occurs on a fixed schedule regardless of the bridge condition. Hence, damage could go undetected for a long period if it occurs early within the period between bi-annual inspections. Finally, visual inspections introduce some degree of subjectivity in the inspection process. For example, a recent study in the United Sates by the Federal Highway Administration (FHWA) reported significant variability in the ratings assigned by a cohort of highly trained inspectors [5].

Bridge management practices can clearly benefit from many of the new sensing technologies emerging from the engineering discipline. Sensors permanently installed in a bridge offer two distinct advantages over visual inspections. First, they create data that can be used to provide bridge managers with a more objective basis for their decision-making. Second, sensors are able to continuously record the response of the bridge.

Statistical methods applied to continuous data sets can provide a probabilistic foundation for accurate risk assessment. In addition, continuous monitoring of bridge behavior allows the *onset* of damage to be detected. If caught early, damage is relatively cheaper to repair than if it is caught after it has grown into a serious state. While the benefits of structural monitoring are widely known, few bridges in the United States actually have monitoring systems installed. The lack of adoption can be attributed to historically high costs associated with the installation of tethered structural monitoring systems. Furthermore, computational tools necessary to process measurement data for indications of structural deterioration and damage lag in maturity compared to sensing technologies. Without robust data interrogation tools autonomously screening measurement data for indications of structural distress, data repositories grow without the data being thoroughly investigated.

Wireless sensing technology has matured to a point that it can now serve as a substitute for wired sensors. Excitement surrounds the emergence of wireless sensors because the technology fundamentally solves the cost and data management problems that have hindered widespread adoption of tethered structural monitoring systems [6-8]. By eliminating the need to install extensive wiring within a large structure, wireless sensors are an order of magnitude cheaper than wired sensors [9]. Furthermore, most wireless sensors include a microcontroller in their design which allows them to locally interrogate raw sensor data [7,10]. Sensor-based data interrogation offers a means of screening raw data for indications of structural distress. In-network data processing provides benefits such as scalable data management (by avoiding raw data inundation at the repository), reduction of communication requirements (since low-bandwidth processed results can be sent in lieu of high-bandwidth raw data streams), and reduction of the energy requirements of the wireless sensor node (since data processing is more energy efficient than use of the wireless radio [11]). While great strides have been made in improving the performance and reliability of wireless sensors, full-scale field testing of the technology is critical to ensuring further development and maturity.

To validate the performance of emerging structural health monitoring technologies, an international collaboration between researchers in the United States and Korea has been established for testing structural health monitoring system components within operational bridges. The US-Korea collaboration is focused on installing novel sensors upon three highway bridges situated along a 7.7 km long test road constructed and managed by the Korea Expressway Corporation. In this study, a wireless sensor prototype under development at the University of Michigan is installed within two of the three bridges. The reliability of the wireless monitoring systems is assessed while the quality of the wireless sensor data is compared to equivalent data derived from a traditional wired monitoring system. Finally, the modal characteristics of the two bridges are identified using the wireless monitoring system data.

2. Testbed Bridges: A redundant section of the Jungbu Inland Expressway was constructed by the Korea Expressway Corporation in 2002 outside of Icheon, Korea. The test road spans 7.7 km and carries two lanes of south-bound traffic. The road is instrumented with a variety of sensors (including pressure sensors, strain gages, and thermometers); this instrumentation is intended to monitor the long-term behavior of roads designed by current Korean design codes. To carry highway traffic over irrigation valleys that service the agricultural region, three highway bridges exist along the test road: the Geumdang, Yeondae and Samseung Bridges. These three bridges are not instrumented with sensors since they fall outside of the purview of the Korea Expressway Corporation pavement study.

The Geumdang Bridge (Fig. 1) is the northern-most bridge along the test road. The bridge has a total span of 273 m and is 12.6 m wide; in addition, it has a 15º skew. The 151 m long northern span is constructed from four pre-cast concrete girders with a 27 cm thick concrete deck. The 122 m long southern span is constructed as a continuous concrete box girder whose height is 2.6 m. The box girder is supported at its two ends by a concrete pier and an abutment structure. Two additional concrete piers provide support along the interior spans of the box girder span.

The Yeondae Bridge (Fig. 2) spans 180 m making it a shorter bridge compared to the Geumdang Bridge. The bridge section employs two identical trapezoidal steel box girders that are each 2.2 m tall, 3.3 m wide at the top, and 2.1 m at the bottom. While abutment structures support the two ends of the bridge, three concrete piers are located every 45 m along the length of the bridge. The geometry of the bridge is more complicated than the Geumdang Bridge because of a relatively large (40º) skew angle and curved plan geometry.

The Samseung Bridge (Fig. 3) is the shortest of the three test road bridges with a span of 46 m. The bridge supports roadway traffic using 5 built-up steel I-beam sections. A 30 cm concrete deck is constructed to be in composite action with the steel girders. Along the

Figure 1: Geumdang Bridge

Figure 2: Yeondae Bridge

Figure 3: Samseung Bridge

length of the bridge, beams and braced stiffener elements are installed between the load carrying steel girders to provide lateral support.

In this study, the Geumdang and Yeondae Bridges are selected for modal analysis. To record the behavior of the two bridges, a wireless monitoring system was installed in each bridge. Each wireless monitoring system consisted of wireless sensor nodes with a microelectromechanical system (MEMS) accelerometer interfaced to each node. Controlled truck loading was employed to excite each bridge. During testing, the test road was closed by the Korea Expressway Corporation and one calibrated truck was given access to the road. The truck was driven over each bridge at a fixed speed so as to introduce structural vibrations.

3. Wireless Structural Monitoring: The Geumdang and Yeondae Bridges were instrumented separately on two different occasions. The Geumdang Bridge was instrument in July 2005 while the Yeodae Bridge was instrumented three years later in July 2008. Given the time separation between the two measurement campaigns, different wireless sensor prototypes were employed. The Geumdang Bridge was instrumented with a wireless monitoring system developed at Stanford University termed *WiMMS*. However, the Yeondae Bridge was instrumented with the *Narada* wireless monitoring system currently under development at the University of Michigan.

3.1 Stanford WiMMS Wireless Sensor Prototype: The Stanford WiMMS wireless sensor node (Fig. 4) was developed specifically for structural health monitoring applications. Key attributes of the wireless sensor are a high-resolution analog-to-digital converter (ADC) for the collection of ambient vibration measurements and far-reaching inter-nodal communication ranges properly scaled to the dimensions of civil infrastructure systems. The hardware design of the WiMMS wireless sensor includes the Texas Instruments ADS8341 4-channel 16-bit ADC. This specific ADC is capable of sampling at data rates in excess of 100 kHz. Once data is collected by the 4-channel ADC, it is passed to the Atmel ATmega 128 microcontroller. This microcontroller has ample on-chip flash memory (128 kB) for the storage of embedded software. However, the scarce on-chip random access memory requires that 128 kB of off-chip RAM be included in the wireless sensor for data storage. For wireless communications, the MaxStream 9XCite radio operating on the 900 MHz radio spectrum is integrated in the wireless sensor design. This radio has a 300 m communication range and a 38.4 kilobit per second data rate. The wireless sensor's active power consumption is 380 mW but it only consumes 0.5 mW when powered down into a sleep mode [12].

3.2 Michigan Narada Wireless Sensor Prototype: The Narada wireless sensor (Fig. 5) is a modified version of the WiMMS wireless sensor. For example, the Narada wireless sensor utilizes the same ADC (Texas Instruments ADS8341), microcontroller

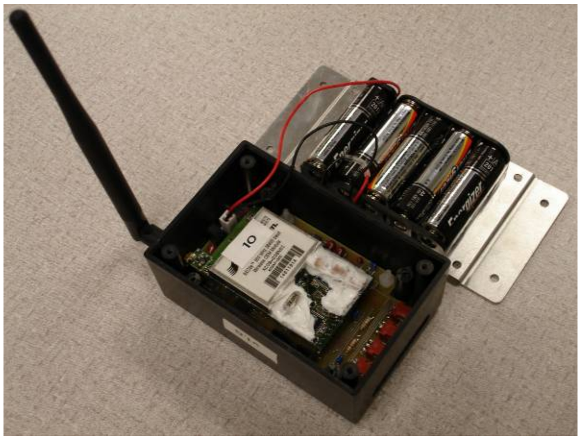

Figure 4: WiMMS wireless sensor node

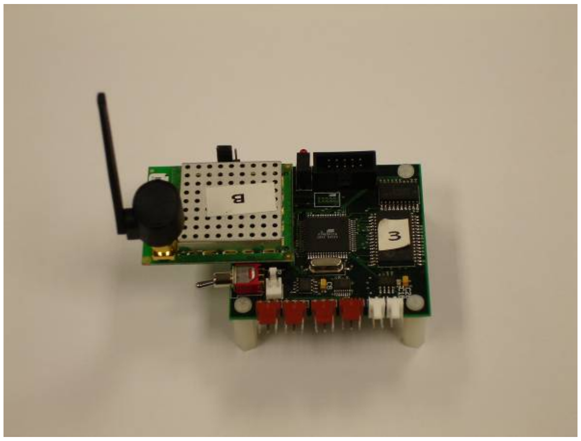

Figure 5: Narada wireless sensor node

(Atmel ATmega 128), and off-chip RAM chip as the WiMMS wireless sensor. However, an entirely new (and lower power) wireless radio is selected for inclusion in the Narada wireless sensor. The Chipcon CC2420 transceiver operating on the 2.4 GHz radio band is selected. This specific radio operates using the IEEE 802.15.4 wireless communication standard; therefore it can freely communicate with any other wireless sensor node that uses the IEEE 802.15.4 communication standard. The nominal range of the wireless sensor radio is roughly 30 m which is quite small for large-scale structures. Hence, a power-amplified version of the Chipcon CC2420 has been custom designed to provide communication ranges in excess of 300 m. An additional functional feature added to Narada is a 2 channel, 12-bit digital-to-analog converter (Texas Instruments DAC7612). The DAC is included for feedback control of civil structures using semi-active control devices [13]. The active power consumption of the wireless sensor is roughly 250 mW which is lower than the WiMMS wireless sensor, due in large part to the more efficient long-range radio in the Narada wireless sensor design.

4. Geumdang Bridge Monitoring: The Geumdang Bridge was instrumented with a network of 14 WiMMS wireless sensors configured to operate as a wireless monitoring system. Interfaced to each wireless sensor node was a low-cost MEMS accelerometer. The PCB Piezotronics 3801D1FB3G accelerometer was selected because of its low cost (roughly $250 per sensor), moderate sensitivity (0.7 V/g), and adequate dynamic range (± 3g). With a 150 μg noise floor, the accelerometer is also capable of accurate measurement of structural ambient vibrations which are characterized by low acceleration amplitudes. Due to quantization noise inherent to the wireless sensor ADC, the voltage output of the PCB 3801D1FB3G was amplified using a custom-made signal amplification circuit; an amplification factor of 20 was selected [14].

The 14 wireless sensors are installed along the length of the concrete box girder of the Geumdang Bridge. The accelerometers are mounted to level plates on the interior of the box girder to ensure perfect orientation in the vertical direction. The location of the wireless sensors with their accelerometers and signal amplification circuits are denoted in Fig. 6. The wireless sensors were evenly distributed across the box girder to ensure accurate operational deflection shapes could be calculated from the measurement data. In the center of the bridge was a laptop computer that served as the coordinator of the wireless monitoring system. Furthermore, all wireless sensor data was transmitted to the coordinator for permanent storage and off-line analysis

In addition to the wireless sensors, traditional piezoelectric accelerometers (PCB Piezotronics 393B12) were installed in the bridge and interfaced to a cable-based monitoring system. The PCB 393B12 accelerometers are seismic-grade piezoelectric accelerometers with a ±0.5 g dynamic range, high sensitivity (10 V/g), and an impressively low noise floor (8 μg). As shown in Fig. 6, the majority of the piezoelectric accelerometers were collocated with the wireless sensor nodes. The piezoelectric accelerometers were interfaced to a 16-channel PCB Piezotronics 481A03 signal conditioner unit using shielded coaxial wires. Sensor outputs were amplified by a factor of 100 before being sampled by a National Instruments 6062E 12-bit data acquisition system. The signal conditioning and data acquisition units were placed at the southern abutment of the bridge.

The bridge was excited using a 4 axel truck calibrated to a weight of 40 metric tons. The truck was driven

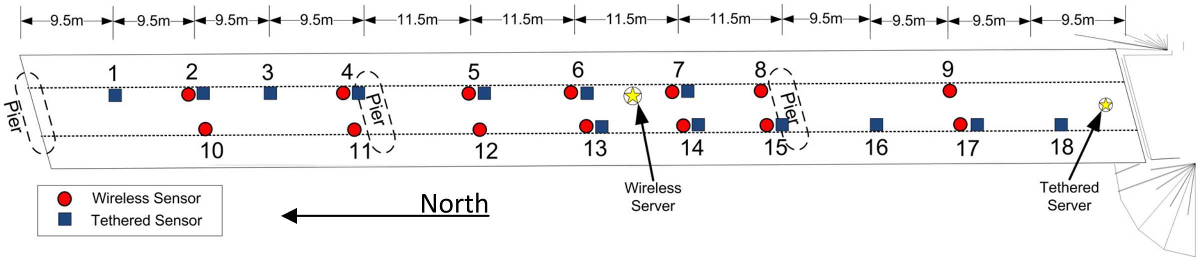

Figure 6: Configuration of wireless and wired accelerometers installed on the Geumdang Bridge during field testing in July 2005.

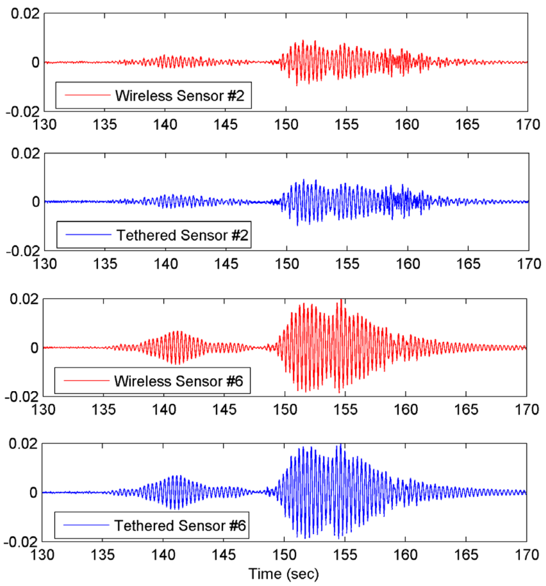

Figure 7: Acceleration time history measurements (in g's) of the Geumdang Bridge at sensor locations #2 and #6. Both wireless and wired response histories are presented for comparison purposes.

across the bridge in a southern direction at set speeds. Both monitoring systems were commanded to record the acceleration response of the bridge prior to the truck loading the bridge. During data collection, both monitoring systems were commanded to record data using 200 Hz sample rates.

Typical acceleration time history responses recorded at sensor locations #2 and 6 are presented in Fig. 7. Sensor location #2 is at the center of the first span while sensor location #6 is near the center of the second span. The plotted responses correspond to the truck crossing the bridge at a speed of 40 km/hr. When comparing the wireless and wired time history records, excellent agreement is found. This confirms that the wireless monitoring system provides high-precision measurements on par with those from a commercial-grade data acquisition system.

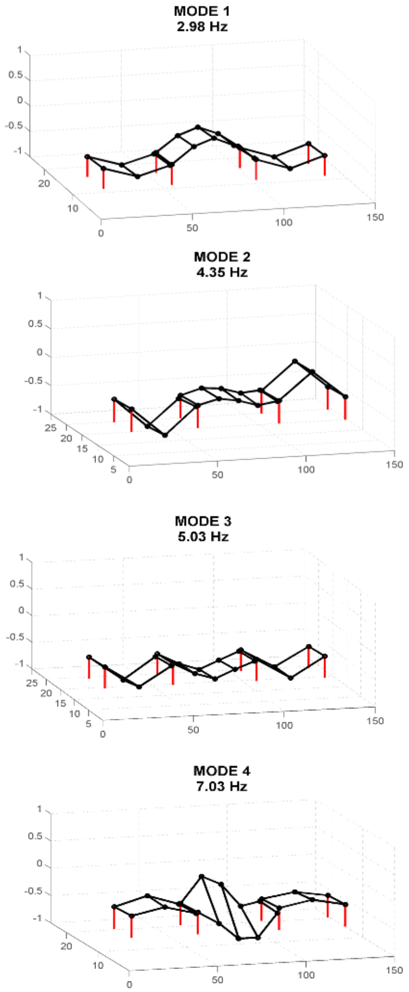

Figure 8: First four operational deflection shapes of the Geumdang Bridge estimated by FDD modal analysis.

Using the time history measurements recorded by the wireless monitoring system, modal analysis of the bridge was conducted. Specifically, traditional peak picking in the frequency domain was adopted to identify the modal frequencies of the structure. The identified modal frequencies were 2.98, 4.35, 5.03 and 7.03 Hz. In addition, the operational deflection shapes

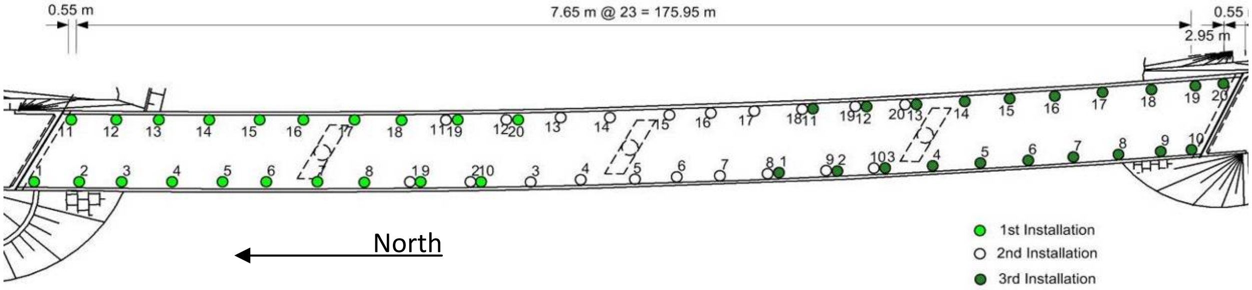

Figure 9: Configuration of the wireless accelerometers installed on the Yeondae Bridge during field testing in July 2008. Three network configurations are adopted for 20 Narada wireless sensors to yield 50 unique measurement points.

of the bridge were estimated using the frequency domain decomposition (FDD) modal estimation technique [15]. Because the FDD technique was applied using response data corresponding to forced vibrations during a narrow-band, non-stationary excitation source (*i.e.,* speeding truck), shapes derived are operational deflection shapes and technically not mode shapes. However, the operational deflection shapes are well correlated to the mode shapes of the bridge. The first four operational deflection shapes estimated by FDD are presented in Fig. 8.

5. Yeondae Bridge Monitoring: A wireless monitoring system assembled from 20 Narada wireless sensors was installed on the top surface of the Yeondae Bridge in July 2008. Crossbow CXL02 MEMS accelerometers were interfaced to 14 Narada wireless sensors while PCB 3801D1FB3G accelerometers were interfaced to the remaining 6 Narada nodes. The Crossbow CXL02 accelerometer offers a level of performance similar to the PCB 3801D1FB3G; for example, the sensitivity, dynamic range and noise floor of the CXL02 is 1 V/g, + 2g, and 150 μg, respectively. To amplify the output of both accelerometers, a signal amplification circuit that amplified the signal by a factor of 20 was integrated with each wireless node-accelerometer pair. All accelerometers were mounted to the road surface oriented to measure vertical acceleration of the deck. Unlike the Geumdang Bridge, a wired structural monitoring system was not installed in the Yeondae Bridge during the wireless measurement campaign.

Given the complex geometry of the Yeondae Bridge (*e.g.,* large skew angle, curved plan), a dense instrumentation strategy was desired. With the monitoring system being assembled from wireless sensor nodes, the system topology could be reconfigured by physically moving wireless sensors. During field testing, the wireless monitoring system was reconfigured twice as shown in Fig. 9. In total, three network configurations were adopted to achieve 50 unique measurement points along the bridge length. First, Narada wireless sensor nodes were installed on the east and west sides of the bridge congregated near the northern abutment. After data was collected, the 20 Narada-accelerometer pairs were physically moved to the center of the bridge. Finally, the third installation configuration was adopted with Narada wireless sensor nodes congregated at the southern end of the bridge. To ensure continuity between the three network configurations, intentional overlap was adopted between the different network configurations; specifically, four measurement points were in common between each network configuration. Network reconfiguration was a relatively easy task taking approximately one hour to move all of the wireless sensor nodes.

A three axel truck with a calibrated weight of 25 metric tons was utilized to introduce vibrations into the Yeondae Bridge. The truck was driven in a southern direction at fixed speeds ranging from 30 to 70 km/hr. During each run of the truck, the wireless monitoring system was commanded by a laptop computer located at the northern abutment to collect acceleration response data at a 100 Hz sample rate. The wireless sensors were commanded to record 90 seconds of data before transmitting their time histories back to the laptop. Given the short time of collection, the wireless monitoring system initiated data collection shortly before the truck entered the bridge. The time window was long enough that the free vibration response of the bridge was captured along with the forced response.

A typical time history acceleration response recorded at sensor location #20 in the first system installation is presented in Fig. 10. As can be seen, the point in time at which the truck entered the bridge is easy to identify (roughly at 16 sec). Based on the speed of the truck (30 km/hr), the time at which the truck exited the bridge can also be estimated. The peak acceleration response

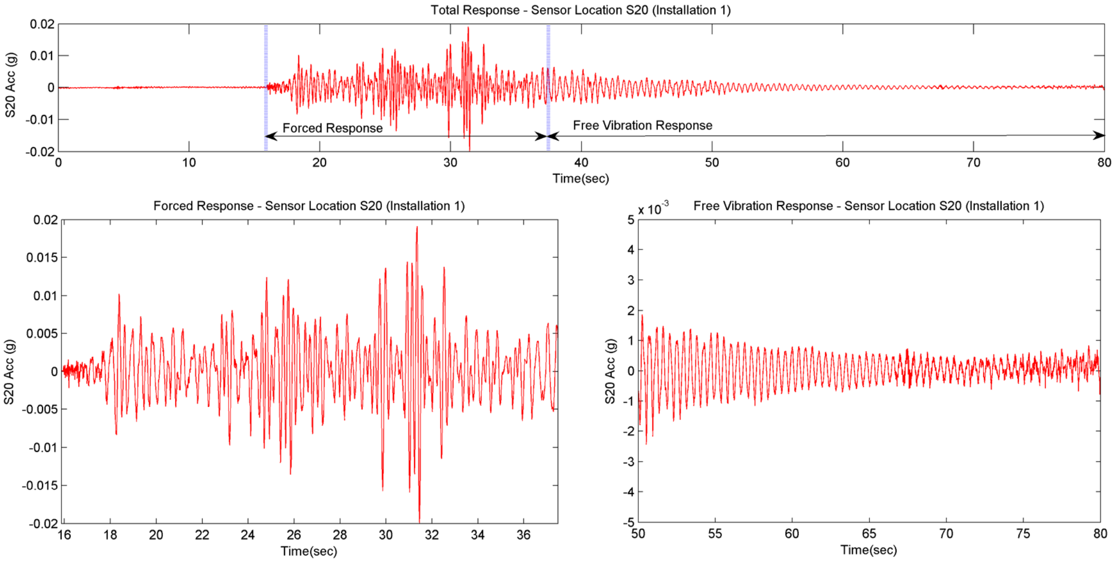

Figure 10: Time history acceleration response of the Yeondae Bridge recorded at sensor location #20 (during the first system installation). The 25 metric ton truck crossed the bridge at a speed of 30 km/hr. The dotted lines in the top figure show when the truck entered and exited the bridge. The forced (lower left) and free (lower right) vibration response is enlarged to provide insight to the data quality.

of the bridge occured shortly after 31 seconds with a maximum absolute bridge response of 20 mg obtained.

Free vibration time history responses were converted to the frequency domain through the discrete Fourier transform. Peak picking using the output spectra was employed to identify the primary modal frequencies of the structure: 2.25, 2.64, 3.47, 4.05, 4.93 Hz. For each of the network installations, the operational deflection shapes of the bridge were calculated from the output-only data set using the FDD modal estimation technique. The operational deflection shapes calculated correspond to each sensor installation; to derive the global operational deflection shapes, the shapes determined for each network installation were stitched together. When scaling operational deflection shapes during the stitching process, a scaling factor that minimizes the absolute difference between the operational deflection shapes at the 4 common nodal points was used. The first five operational deflection shapes of the Yeondae Bridge are presented in Fig. 11. The first four operational deflection shapes correspond to flexural modes while the fifth shape is attributed to the primary torsion mode of the bridge.

6. Conclusions: In this study, wireless monitoring systems assembled from low-cost wireless sensor prototypes (WiMMS and Narada) were installed on the Geumdang and Yeondae Bridges in Korea. Both bridges currently serve as testbed bridges at the center of an ongoing U.S.-Korea international collaboration focused on new sensing technologies for structural health monitoring of civil infrastructure. In the Geumdang Bridge, 14 WiMMS wireless sensor nodes were installed in a fixed configuration. In contrast, 20 Narada wireless sensors were installed in the Yeondae Bridge in three different network configurations to yield a total of 50 independent measurement points along the bridge length. The vertical acceleration response of both bridges were captured during forced excitation using trucks of a known calibrated weight driving across the bridges at fixed speeds ranging from 30 to 70 km/hr.

For both bridges, reliable performance of the wireless monitoring system was encountered. For example, a robust send-acknowledgement communication between the wireless sensors and the data repository ensured a 100% success rate in data delivery. The quality of the measured accelerations was equivalent to the accelerations recorded by the tethered structural monitoring system. The high-quality wireless time history records were also used to derive the modal frequencies and operational deflection shapes of both bridges. A combination of reasonable flexural and torsional operational deflection shapes was obtained for both bridges.

The development of wireless structural health monitoring systems is an on-going research endeavor. Current research efforts are focused on the permanent installation of a Narada-based wireless monitoring system within the Geumdang and Yeondae Bridges. Furthermore, embedded data processing by the wireless

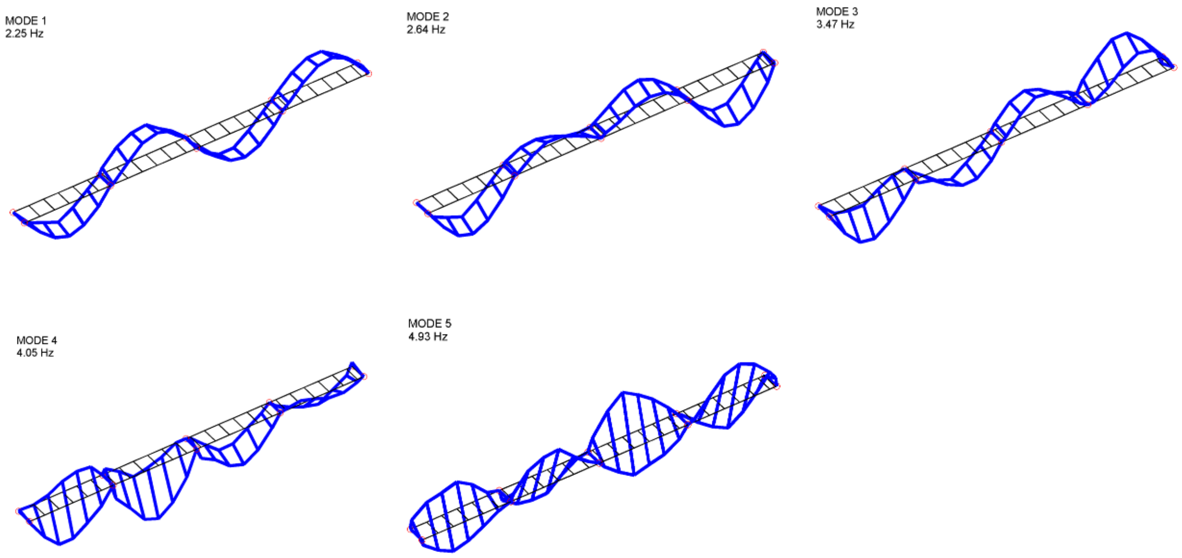

Figure 11: First five operational deflection shapes of the Yeondae Bridge estimated by FDD modal analysis.

monitoring system nodes is being explored for automated modal analysis and damage detection of the Geumdang and Yeondae Bridges.

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