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Title: *Substructure Model Updating through Modal Dynamic Residual Approach*

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ABSTRACT

Difference usually occurs between predictions of a finite element (FE) model and experimental measurements from the actual structure. To achieve more accurate predictions, FE models can be updated based on field measurement data. In the past few decades, numerous FE model updating methods have been developed. Nevertheless, when applied to a large structure, most methods are computationally challenging and expensive, because these methods operate on a complex model for the entire structure. To address this issue, a substructure-based updating method is presented in this paper. The Craig-Bampton theory is adopted to condense the entire structural model into a substructure (currently being analyzed) and a residual structure. Dynamic response of the residual structure is approximated using only a limited number of dominant mode shapes. A modal dynamic residual approach is adopted for updating the substructure model. This substructure model updating procedure (through modal dynamic residual approach) is validated by numerical simulation with a 200 degrees-of-freedom spring-mass model. The performance is compared with an updating procedure based on a conventional modal property difference approach.

INTRODUCTION

Although significant advances have been achieved in finite element (FE) modeling over the past decades, it is practically infeasible to generate a numerical model that behaves exactly the same as a large structure in the field. Predictions by FE models often differ from experimental results, due to the limited accuracy of FE modeling and complexity of the actual structures. For example, many simplifications are adopted in FE modeling, such as idealized hinges or rollers, whereas the simplifications introduce discrepancies from reality. Besides, FE models are generally built based on design drawings years ago, while the actual constructed structure was not perfectly identical as in initial drawings, and may have deteriorated over time.

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Therefore, updating FE models based on field measurements is an essential means to obtain a more accurate structural model.

In the past few decades, various FE model updating methods have been developed and practically applied [1]. Many of these methods utilize modal analysis results from field testing. Selected structural parameters are updated by solving an optimization problem. One major category of model updating methods minimizes the difference between experimental and simulated modal properties [2-4]. This category will be referred as modal property difference approach. Another category of model updating methods will be referred as modal dynamic residual approach, which minimizes modal dynamic residuals from the generalized eigenvalue equation involving stiffness and mass matrices [5-7]. Nevertheless, previous methods generally suffer computational difficulties while updating the model of a large-scale structure with dense measurements, because the methods usually operate on the entire structural model that can have a large number of degrees of freedom (DOFs).

In order to address the computational difficulty, particularly to accommodate data collected at dense measurement locations, substructure-based FE model updating has been investigated. A well-known substructure modeling method is the Craig-Bampton theory that partitions a large structure into a substructure being analyzed and a residual structure containing the rest of the DOFs [8]. Dynamic response of the residual structure is approximated using only a limited number of dominant mode shapes, so that the large structural model is condensed to a simplified model with much smaller number of DOFs. The updating of such a sub/residual-structure model has been studied using modal property difference approach [9].

In comparison, this research investigates substructure model updating through modal dynamic residual approach. The updating performance is compared with the previous updating method based on conventional modal property difference approach. The rest of the paper is organized as follows. The formulations of substructure modeling and updating using modal dynamic residual approach are presented first. Numerical investigation on a 200-DOF spring-mass structure is then described. The performance of the presented approach is compared with the updating procedure based on conventional modal property difference approach. Finally, a summary and discussion are provided.

SUBSTRUCTURE MODELING

Figure 1 illustrates the substructure modeling strategy following Craig-Bampton theory [8]. Subscripts s, i, and r are used to denote the substructure being analyzed, the interface nodes, and the residual structure, respectively. The block-bidiagonal structural stiffness and mass matrices, $\mathbf{K}$ and $\mathbf{M}$, can be assembled using original DOFs $\mathbf{x} = \begin{bmatrix} \mathbf{x}_s & \mathbf{x}_i & \mathbf{x}_r \end{bmatrix}^{\mathrm{T}}$:

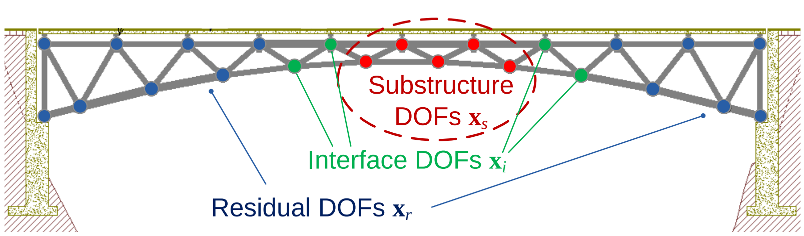

Figure 1. Illustration of substructure modeling strategy.

$$\mathbf{K}=\begin{bmatrix}\begin{bmatrix} & \\ & \mathbf{K}_S & \\ & \end{bmatrix} & \mathbf{0}\\ & \mathbf{0}\\ \mathbf{0} \quad \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{0} \quad \mathbf{0}\\ \mathbf{0} & \begin{bmatrix} & \\ & \mathbf{K}_R \\ & \end{bmatrix}\\ \mathbf{0} & \end{bmatrix}=\begin{bmatrix}\begin{bmatrix}\mathbf{K}_{ss} & \mathbf{K}_{si}\\ \mathbf{K}_{is} & \mathbf{K}_{ii}^{S}\end{bmatrix} & \mathbf{0}\\ & \mathbf{0}\\ \mathbf{0} \quad \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{0} \quad \mathbf{0}\\ \mathbf{0} & \begin{bmatrix}\mathbf{K}_{ii}^{R} & \mathbf{K}_{ir}\\ \mathbf{K}_{ri} & \mathbf{K}_{rr}\end{bmatrix}\\ \mathbf{0} & \end{bmatrix} \tag{1}$$

$$\mathbf{M}=\begin{bmatrix}\begin{bmatrix} & \\ & \mathbf{M}_S & \\ & \end{bmatrix} & \mathbf{0}\\ & \mathbf{0}\\ \mathbf{0} \quad \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{0} \quad \mathbf{0}\\ \mathbf{0} & \begin{bmatrix} & \\ & \mathbf{M}_R \\ & \end{bmatrix}\\ \mathbf{0} & \end{bmatrix}=\begin{bmatrix}\begin{bmatrix}\mathbf{M}_{ss} & \mathbf{M}_{si}\\ \mathbf{M}_{is} & \mathbf{M}_{ii}^{S}\end{bmatrix} & \mathbf{0}\\ & \mathbf{0}\\ \mathbf{0} \quad \mathbf{0} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{0} \quad \mathbf{0}\\ \mathbf{0} & \begin{bmatrix}\mathbf{M}_{ii}^{R} & \mathbf{M}_{ir}\\ \mathbf{M}_{ri} & \mathbf{M}_{rr}\end{bmatrix}\\ \mathbf{0} & \end{bmatrix} \tag{2}$$

where $\mathbf{K}_S$ and $\mathbf{M}_S$ denote the stiffness and mass matrices of the substructure; $\mathbf{K}_R$ and $\mathbf{M}_R$ denote the stiffness and mass matrices of the residual structure; $\mathbf{K}_{ii}^{S}$ and $\mathbf{M}_{ii}^{S}$ denote the stiffness and mass entries of the interface DOFs and contributed by the substructure; $\mathbf{K}_{ii}^{R}$ and $\mathbf{M}_{ii}^{R}$ denote the stiffness and mass entries of the interface DOFs and contributed by the residual structure.

The dynamic behavior of the residual structure can be approximated using Craig-Bampton formulation [8]. The DOFs of the residual structure, $\mathbf{x}_r \in \mathbb{R}^{n_r}$, are approximated by a linear combination of interface DOFs, $\mathbf{x}_i \in \mathbb{R}^{n_i}$, and modal coordinates of the residual structure, $\mathbf{q}_r \in \mathbb{R}^{n_q}$.

$$\mathbf{x}_r \approx \mathbf{T}\mathbf{x}_i + \mathbf{\Phi}_r \mathbf{q}_r \tag{3}$$

Here $\mathbf{T} = -\mathbf{K}_{rr}^{-1}\mathbf{K}_{ri}$ is the Guyan static condensation matrix; $\mathbf{\Phi}_r = \left[\boldsymbol{\varphi}_1, \ldots, \boldsymbol{\varphi}_{n_q}\right]$ represents the mode shapes of the residual structure fixed at the interface.

$$\left(-\omega_{r,j}{}^2\mathbf{M}_{rr} + \mathbf{K}_{rr}\right)\boldsymbol{\varphi}_j = 0, \qquad j = 1, \ldots, n_q \tag{4}$$

Although the size of the residual structure may be large, the number of modal coordinates, n_q, can be chosen as relatively small to reflect the first few dominant mode shapes only (i.e. $n_q << n_r$). The transformation can be written in vector form:

$$\begin{bmatrix}\mathbf{x}_i\\ \mathbf{x}_r\end{bmatrix} \approx \mathbf{\Gamma}\begin{bmatrix}\mathbf{x}_i\\ \mathbf{q}_r\end{bmatrix}, \text{ where } \quad \mathbf{\Gamma} = \begin{bmatrix}\mathbf{I} & \\ \mathbf{T} & \mathbf{\Phi}_r\end{bmatrix} \tag{5}$$

Suppose $\tilde{\mathbf{K}}_R$ and $\tilde{\mathbf{M}}_R$ denote the new stiffness and mass matrices of the residual structure after transformation:

$$\tilde{\mathbf{K}}_R = \mathbf{\Gamma}^{\mathrm{T}}\mathbf{K}_R\mathbf{\Gamma} \qquad\qquad \tilde{\mathbf{M}}_R = \mathbf{\Gamma}^{\mathrm{T}}\mathbf{M}_R\mathbf{\Gamma} \tag{6}$$

Link [9] proposed a model updating method for matrices of both the substructure and the residual structure, where the substructure model is updated as

$$\mathbf{K}_S = \mathbf{K}_{S0} + \sum_{j=1}^{n_\alpha}\alpha_j\mathbf{K}_{S0,j} \qquad \mathbf{M}_S = \mathbf{M}_{S0} + \sum_{j=1}^{n_\beta}\beta_j\mathbf{M}_{S0,j} \tag{7}$$

where $\mathbf{K}_{S0}$ and $\mathbf{M}_{S0}$ are the stiffness and mass matrices of the substructure and used as initial starting point in the model updating; α_j and β_j correspond to physical system parameters to be updated, such as elastic modulus and density of each element; n_α and n_β represent the total number of updating system parameters; $\mathbf{K}_{S0,j}$ and $\mathbf{M}_{S0,j}$

are constant matrices determined by the type and location of these physical parameters. For the rest of this paper, subscript "0" will be used to denote variables associated with the initial structural model, which serves as the starting point for model updating.

Assuming that physical parameter changes in the residual structure do not alter the mode shapes significantly, the transformed residual structural model is updated by

$$\tilde{\mathbf{K}}_R = \tilde{\mathbf{K}}_{R0} + \sum_{j=1}^{n_q+n_i} p_j \tilde{\mathbf{K}}_{R0,j} \qquad \tilde{\mathbf{M}}_R = \tilde{\mathbf{M}}_{R0} + \sum_{j=1}^{n_q+n_i} q_j \tilde{\mathbf{M}}_{R0,j} \tag{8}$$

where p_j and q_j are modal parameters to be updated; $\tilde{\mathbf{K}}_{R0}$ and $\tilde{\mathbf{M}}_{R0}$ are the initial stiffness and mass matrices of the transformed residual structure model; $\tilde{\mathbf{K}}_{R0,j}$ and $\tilde{\mathbf{M}}_{R0,j}$ represent the constant correction matrices formulated using modal back-transform [9]. Detailed formulation can also be found in [10].

Using the model matrices to be updated, i.e. Eq. (7) for substructure and Eq. (8) for residual structure, the entire structural model with reduced DOFs $\begin{bmatrix} \mathbf{x}_s & \mathbf{x}_i & \mathbf{q}_r \end{bmatrix}^{\mathrm{T}}$ can be updated with variables α_j, β_j, p_j, and q_j. For brevity, these variables will be referred to in their vector forms as $\boldsymbol{\alpha} \in \mathbb{R}^{n_\alpha}$, $\boldsymbol{\beta} \in \mathbb{R}^{n_\beta}$, $\mathbf{p} \in \mathbb{R}^{n_q+n_i}$ and $\mathbf{q} \in \mathbb{R}^{n_q+n_i}$.

$$\tilde{\mathbf{K}} = \tilde{\mathbf{K}}_0 + \sum_{j=1}^{n_\alpha} \alpha_j \begin{bmatrix} \begin{bmatrix} \mathbf{K}_{S0,j} \end{bmatrix} & \mathbf{0} \\ & \mathbf{0} \\ \mathbf{0} \quad \mathbf{0} & \mathbf{0} \end{bmatrix} + \sum_{j=1}^{n_i+n_q} p_j \begin{bmatrix} \mathbf{0} & \mathbf{0} \quad \mathbf{0} \\ \mathbf{0} & \\ \mathbf{0} & \begin{bmatrix} \tilde{\mathbf{K}}_{R0,j} \end{bmatrix} \end{bmatrix} = \tilde{\mathbf{K}}_0 + \sum_{j=1}^{n_\alpha} \alpha_j \mathbf{S}_{\alpha,j} + \sum_{j=1}^{n_i+n_q} p_j \mathbf{S}_{p,j} \tag{9}$$

$$\tilde{\mathbf{M}} = \tilde{\mathbf{M}}_0 + \sum_{j=1}^{n_\beta} \beta_j \begin{bmatrix} \begin{bmatrix} \mathbf{M}_{S0,j} \end{bmatrix} & \mathbf{0} \\ & \mathbf{0} \\ \mathbf{0} \quad \mathbf{0} & \mathbf{0} \end{bmatrix} + \sum_{j=1}^{n_i+n_q} q_j \begin{bmatrix} \mathbf{0} & \mathbf{0} \quad \mathbf{0} \\ \mathbf{0} & \\ \mathbf{0} & \begin{bmatrix} \tilde{\mathbf{M}}_{R0,j} \end{bmatrix} \end{bmatrix} = \tilde{\mathbf{M}}_0 + \sum_{j=1}^{n_\alpha} \beta_j \mathbf{S}_{\beta,j} + \sum_{j=1}^{n_i+n_q} q_j \mathbf{S}_{q,j} \tag{10}$$

where $\mathbf{S}_{\alpha,j}$, $\mathbf{S}_{\beta,j}$, $\mathbf{S}_{p,j}$ and $\mathbf{S}_{q,j}$ represent the sensitivity matrices corresponding to variables α_j, β_j, p_j, and q_j, respectively.

MODAL DYNAMIC RESIDUAL APPROACH

To update the substructure model, a modal dynamic residual approach is applied in this study. The modal dynamic residual approach minimizes modal dynamic residuals from the generalized eigenvalue equation.

$$\underset{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{p},\mathbf{q}}{\text{Minimize}} \qquad \sum_{j=1}^{n_{\text{meas}}} \left\| \left(\tilde{\mathbf{K}} - \omega_{\text{exp},j}^2 \tilde{\mathbf{M}} \right) \boldsymbol{\psi}_{\text{exp},j} \right\|^2 \tag{11}$$

where $\|\bullet\|$ denotes Euclidean norm (2-norm); $\omega_{\text{exp},j}$ and $\boldsymbol{\psi}_{\text{exp},j}$ denote the j-th modal frequency and mode shape from experimental data; $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\mathbf{p}$, and $\mathbf{q}$ are the system parameters to be updated; n_{meas} denotes the number of measured modes.

During dynamic testing in the field, usually not all DOFs can be measured by sensors. Besides, the mode shapes corresponding to the generalized coordinates $\mathbf{q}_r$ cannot be physically measured. This indicates that only incomplete mode shapes can be directly obtained from experimental data. Therefore, in addition to updating parameters, part of $\boldsymbol{\psi}_{\exp,j}$ is unknown in the optimization formulation in Eq. (11). An iterative linearization procedure for efficiently solving the optimization problem is proposed in [5]. Each iteration includes two steps:

Step (i) Modal Expansion

In Step (i), updating parameters ($\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\mathbf{p}$, and $\mathbf{q}$) are treated as constant. The parameter values are either based on initial estimation, or from model updating results in the last iteration. The unknown part of each experimental mode shape vector $\boldsymbol{\psi}_{\exp,j}$ is obtained by solving the optimization problem (Eq. (11)) in least square form. To lighten notation, $\boldsymbol{\psi}_{\exp,j}$ is simplified as $\boldsymbol{\psi}_j$ in the rest of this section. The modal expansion is performed as:

$$\boldsymbol{\psi}_{j,U} = -\left(\mathbf{A}_{j,UU}^{-1}\mathbf{A}_{j,UM}\right)\boldsymbol{\psi}_{j,M} \tag{12}$$

where subscripts M and U represent the measured and unmeasured DOFs, respectively; $\boldsymbol{\psi}_{j,M}$ and $\boldsymbol{\psi}_{j,U}$ represent the measured and unmeasured entries of the j-th mode shape vector. Terms in the expansion matrix $\left(\mathbf{A}_{j,UU}^{-1}\mathbf{A}_{j,UM}\right)$ come from the generalized eigenvalue problem in structural dynamics.

$$\begin{bmatrix} \mathbf{A}_{j,MM} & \mathbf{A}_{j,MU} \\ \mathbf{A}_{j,UM} & \mathbf{A}_{j,UU} \end{bmatrix} = \left[\mathbf{A}_j\right] = \left(\tilde{\mathbf{K}} - \omega_j^2\tilde{\mathbf{M}}\right)^{\mathrm{T}}\left(\tilde{\mathbf{K}} - \omega_j^2\tilde{\mathbf{M}}\right) \tag{13}$$

Step (ii) Parameter Updating

Using the expanded complete mode shapes from Step (i), the system parameters ($\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\mathbf{p}$, and $\mathbf{q}$) can be updated by solving the optimization problem rewritten in least square form. For brevity, following shows the least-square formulation when only stiffness-related parameters, $\boldsymbol{\alpha}$ and $\mathbf{p}$, need to be updated.

$$\begin{bmatrix} \mathbf{S}_{\alpha,1}\boldsymbol{\psi}_1 & \cdots & \mathbf{S}_{\alpha,n_\alpha}\boldsymbol{\psi}_1 & \mathbf{S}_{p,1}\boldsymbol{\psi}_1 & \cdots & \mathbf{S}_{p,n_i+n_q}\boldsymbol{\psi}_1 \\ \mathbf{S}_{\alpha,1}\boldsymbol{\psi}_2 & \cdots & \mathbf{S}_{\alpha,n_\alpha}\boldsymbol{\psi}_2 & \mathbf{S}_{p,1}\boldsymbol{\psi}_2 & \cdots & \mathbf{S}_{p,n_i+n_q}\boldsymbol{\psi}_2 \\ & & \vdots & & & \\ \mathbf{S}_{\alpha,1}\boldsymbol{\psi}_{n_{\text{meas}}} & \cdots & \mathbf{S}_{\alpha,n_\alpha}\boldsymbol{\psi}_{n_{\text{meas}}} & \mathbf{S}_{p,1}\boldsymbol{\psi}_{n_{\text{meas}}} & \cdots & \mathbf{S}_{p,n_i+n_q}\boldsymbol{\psi}_{n_{\text{meas}}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\alpha} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \left(\omega_1^2\tilde{\mathbf{M}}_0 - \tilde{\mathbf{K}}_0\right)\boldsymbol{\psi}_1 \\ \left(\omega_2^2\tilde{\mathbf{M}}_0 - \tilde{\mathbf{K}}_0\right)\boldsymbol{\psi}_2 \\ \vdots \\ \left(\omega_{n_{\text{meas}}}^2\tilde{\mathbf{M}}_0 - \tilde{\mathbf{K}}_0\right)\boldsymbol{\psi}_{n_{\text{meas}}} \end{Bmatrix} \tag{14}$$

Here $\mathbf{S}_{\alpha,j}$ and $\mathbf{S}_{p,j}$ represent the sensitivity matrices from Eq. (9).

NUMERICAL EXAMPLE

To validate the proposed convex optimization method for substructure model updating, simulation is performed with a 200-DOF spring-mass model. In the initial model (as starting point of model updating), all the mass and spring stiffness values are set identically as 6kg and 35kN/m, respectively. Damage is introduced to this model by reducing 10% of spring stiffness at k_{20}, k_{30}, k_{45}, k_{50}, k_{60}, k_{62}, k_{82}, k_{100}, k_{120}, and k_{150}. Figure 2 shows a conceptual drawing of the 200-DOF spring-mass model. A substructure with DOFs from 41 to 54 is selected for model updating. As a result, DOFs 40 and 55 are interface DOFs and all other DOFs belong to the residual

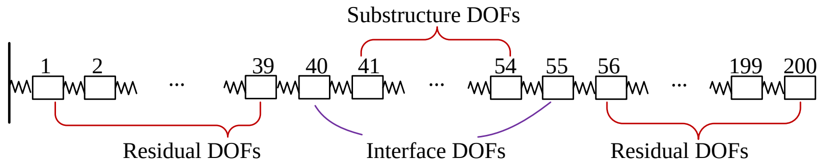

Figure 2. Illustration of substructure selection (10% stiffness reduction is introduced to k_{20}, k_{30}, k_{45}, k_{50}, k_{60}, k_{62}, k_{82}, k_{100}, k_{120}, and k_{150} as damage)

structure. Modal characteristics of the damaged structure are calculated from the mass and stiffness matrices of the damaged structure. For simplicity, the natural frequencies and mode shapes at all substructure and interface DOFs (i.e. DOFs 40 to 55) are directly used as the experimentally measured $\omega_{\exp,j}$ and $\boldsymbol{\psi}_{\exp,j}$ for model updating.

Dynamic response of the residual structure is approximated using twenty modal coordinates, i.e. $n_q = 20$ (Eq. (3)). With $\mathbf{x}_s \in \mathbb{R}^{14\times1}$ and $\mathbf{x}_i \in \mathbb{R}^{2\times1}$, the entire structural model is therefore condensed to 36 DOFs. Note that two springs with stiffness loss, k_{45} and k_{50}, are contained in the substructure. Assuming acceleration measurements are available only on the substructure and interface DOFs, the objective is to identify the damage using proposed substructure updating method. In this example, accurate structural mass matrix is assumed to be known, so $\boldsymbol{\beta}$ and $\mathbf{q}$ are not among the updating parameters. The selected updating parameters are α_1, α_2, ..., α_{15} and p_2, p_3, ..., p_{22}. Parameters α_1, α_2 , ..., and α_{15} denote relative stiffness changes in k_{41}, k_{42}, ..., and k_{55} that belong to the substructure, respectively; p_2, p_3, ..., and p_{22} denote the modal space parameters of the residual structure with free interface. Note that $n_q+n_i = 22$ and that the modal parameter p_1 is not included, because the first resonance frequency of the residual structure with free interface is zero (corresponding to free-body movement). As a result, the first modal correction matrix $\tilde{\mathbf{K}}_{R0,1}$ (Eq. (8)) is a zero matrix [10] .

For comparison, a conventional modal property difference approach for updating the substructure model [9] is also performed. The conventional model updating formulation aims to minimize the difference between experimental and analytical natural frequencies and mode shapes of the substructure model.

$$\underset{\mathbf{u}=\{\boldsymbol{\alpha},\boldsymbol{\beta},\mathbf{p},\mathbf{q}\}^{\mathrm{T}}}{\text{Minimize}} \sum_{j=1}^{n_{\text{meas}}} \left\{ \left(\frac{\omega_{\text{FE},j} - \omega_{\exp,j}}{\omega_{\exp,j}} \right)^2 + \left(\frac{1-\sqrt{\text{MAC}_j}}{\sqrt{\text{MAC}_j}} \right)^2 \right\} \tag{15}$$

where $\omega_{\text{FE},j}$ and $\omega_{\exp,j}$ represent the j-th simulated (from condensed model in Eq. (9) and (10)) and experimental natural frequencies, respectively; MAC_j represents the modal assurance criterion evaluating the difference between the j-th simulated and experimental mode shapes (i.e. between $\boldsymbol{\psi}_{\text{FE},j}$ and $\boldsymbol{\psi}_{\exp,j}$). A nonlinear least-square optimization solver, 'lsqnonlin' in MATLAB toolbox [11], is adopted to numerically solve the optimization problem (Eq. (15)). The optimization solver seeks a minimum through Levenberg-Marquardt algorithm, which adopts a search direction interpolated between the Gauss-Newton direction and the steepest descent direction [12].

Both the modal dynamic residual approach and the conventional modal property difference approach are applied for substructure model updating. For each approach, the updating is performed assuming different numbers of measured modes are available (i.e. modes corresponding to the 1~6 lowest natural frequencies). Table I

Table I. Optimal stiffness changes (%) for substructure elements using modal dynamic residual approach

Stiffness changes	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	Max error
1 mode	0.00	0.00	0.00	0.00	-10.0	0.00	0.00	0.00	0.00	-10.0	0.00	0.00	0.00	0.00	0.00	0.00
2 modes	0.18	0.18	0.17	0.16	-9.86	0.15	0.14	0.13	0.12	-9.90	0.10	0.09	0.08	0.07	0.05	0.18
3 modes	0.27	0.25	0.23	0.21	-9.83	0.17	0.15	0.14	0.12	-9.90	0.09	0.08	0.07	0.06	0.05	0.27
4 modes	0.16	0.15	0.13	0.12	-9.91	0.09	0.08	0.07	0.06	-9.95	0.05	0.04	0.04	0.03	0.02	0.16
5 modes	0.11	0.10	0.09	0.08	-9.93	0.07	0.06	0.06	0.05	-9.96	0.04	0.04	0.03	0.02	0.02	0.11
6 modes	0.05	0.05	0.04	0.04	-9.97	0.03	0.03	0.03	0.03	-9.98	0.02	0.02	0.02	0.01	0.01	0.05

Table II. Optimal stiffness changes (%) for substructure elements using modal property difference approach

Stiffness changes	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	Max error
1 mode	0.87	1.04	0.58	-0.87	-4.37	-0.90	0.40	0.39	-0.90	-4.31	-0.87	0.61	1.17	1.24	0.90	5.69
2 modes	0.20	0.21	-0.23	-1.79	-6.21	-1.72	-0.26	-0.28	-1.76	-6.02	-1.84	-0.29	0.20	0.28	0.10	3.98
3 modes	-0.10	0.03	-0.36	-1.92	-6.73	-1.83	-0.28	-0.27	-1.81	-6.87	-1.91	-0.32	0.08	0.10	-0.16	3.27
4 modes	0.25	0.34	0.39	-0.20	-9.44	-0.15	0.33	0.29	0.00	-9.70	0.03	0.25	0.15	0.17	0.15	0.56
5 modes	0.57	0.91	0.76	-0.64	-7.89	-0.56	0.89	0.88	-0.62	-7.70	-0.70	0.78	1.04	0.95	0.07	2.30
6 modes	0.60	1.32	1.35	1.33	-8.77	1.37	1.42	1.45	1.43	-8.66	1.47	1.54	1.50	1.54	1.73	1.73

summarizes the updating results using the presented modal dynamic residual approach for substructure model updating. When one mode is available, the optimal values for α_5 and α_{10} are -10.0% and -10.0%, which indicate that the stiffness of k_{45} and k_{50} are both reduced by 10.0%. The optimal values for all other α_i are exactly zero, which implies no change in all other stiffness values in the substructure. Therefore, the updating results correctly match the damage locations and severities. With two or more modes measured, the updating results still match the introduced damage well, i.e. the optimal values for α_5 and α_{10} are both very close to -10.0%, and the optimal values for all other α_i are very close to zero. The maximum updating error is only 0.27%. The errors in the cases with two or more modes are mainly caused by the approximations during construction and updating of the substructure model. First, the Craig-Bampton transform used for model condensation (Eq. (3)) neglects interface dynamic contribution and uses only a few dominant modes describing dynamic behavior of residual structure. Second, in order to use constant sensitivity matrices for parameters $\mathbf{p}$ and $\mathbf{q}$ in modal back transform, it is assumed that physical parameter changes in the residual structure do not alter the mode shapes (Eq. (8)). These assumptions may introduce more errors to the updating process when higher modes are involved.

Table II summarizes the updating results using the conventional modal property difference approach. Although α_5 and α_{10} have relatively larger values overall, the damage detection performance is worse than the modal dynamic residual approach. With only one measured mode, the maximum updating error is the largest (5.69%). With more modes measured, the maximum updating error decreases, but the smallest maximum error is 0.56% (with 4 modes), which is still larger than maximum errors using the modal dynamic residual approach. In addition, at locations where stiffness change should be zero, the modal property difference approach overall generates much larger errors than modal residual approach, representing stronger tendency of false positive detection.

SUMMARY AND DISCUSSION

In this paper, a substructure model updating approach is presented. The Craig-Bampton theory is adopted to reduce a large structure model into a smaller model that contains a substructure currently being analyzed and a residual structure. Dynamic response of the residual structure is approximated using only a limited number of dominant mode shapes. A modal dynamic residual approach is applied for updating the substructure model, and an iterative linearization procedure is adopted for efficiently solving the optimization problem. The presented substructure updating method is validated through numerical simulation of a 200-DOF spring-mass model. The updating results successfully detect the damage locations and severities in all cases, when different numbers of measured modes are available. For comparison, a conventional modal property difference approach is applied, and shows lower accuracy than the presented modal dynamic residual approach. Future research will continue to investigate the substructure model updating method on more complicated structural models through numerical simulations and laboratory experiments.

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